Control of the Chaotic Phenomenon in Robot Path using Differential Flatness

Salah Nasr[1], Amine Abadi[1], Kais Bouallegue[2] and Hassen Mekki[1]

[1]*Laboratory of Networked Objects Control and Communication Systems, National Engineering School of Sousse, University of Sousse, Tunisia*

[2]*Department of Electrical Engineering, Higher Institute of Applied Sciences and Technology of Sousse, Tunisia*

Keywords: Chaotic Phenomenon, Mobile Robot, Flatness Control, Chaos Theory.

Abstract: This paper deals with the complex chaotic behavior that can appear in the dynamic trajectory of a mobile robot, when the robot is broken or partly damaged. However, a flatness-based controller is designed to ensure the trajectory planning and tracking. Different mathematical tools have been used such as the flatness control technique and non linear chaotic systems. The simulation results for the kinematic controller are presented to demonstrate the effectiveness of this approach.

1 INTRODUCTION

The control of mobile robots has been the subject of much research in recent years, due to the increasingly frequent use in dangerous or inaccessible environments where human beings can hardly intervene. For autonomous mobile robotics, path generation and execution are very important tasks. Path planning is the process of generating a sequence of trajectory deriving from the assigned task to the mobile robot to be able to perform it.

The general problem is reduced in most cases to move the robot in a known or unknown environment (Belaidi et al., 2017), while avoiding any fixed or mobile obstacles, to carry out a prescribed task. It should define a strategy of movement (path planning)(Hargas et al., 2015), and then execute the prescribed displacement.

The robot controller, which is a major component, has received a lot of attention from researchers. This is why it has a direct impact on its robustness and could prevent its deployment and applicability in several domains (Kumar et al., 2014; Lai, 2014). Many control techniques have been proposed for modern robots including the classical PID, feedback linearization (Korayem et al., 2016; Tinh et al., 2016), inverse dynamics, model predictive control (Klančar and Škrjanc, 2007), adaptive fuzzy-logic control (Bakdi et al., 2017) etc.

Up to now, there has been no experimental work that has treated the chaotic phenomenon in the robot trajectory. On the other hand, the interaction between the theory of chaos and mobile robotics has been only recently studied, as it can be seen in (Nehmzow, 2003), for the generation of the unpredictable trajectory for the robot. For example, the integration between a chaotic system and the robot motion system, dynamic systems, is used to impart the chaotic behavior to a robot like the Arnold system in (Nakamura and Sekiguchi, 2001). An extension of this strategy, applying various chaotic systems on the integration with the kinematics model of the robot, can be found in (Jansri et al., 2004). In (Martins-Filho et al., 2004), the author proposed an open loop control approach to produce unpredictable trajectories so as to control the velocities of the robot wheels, and the state variables of the Lorenz chaotic system are used. Neverthless, there has been no research work to solve the chaotic phenomenon problem that can appear in the robot trajectory.

In this context we propose to use a controller to solve this problem and to facilitate the implementation of our work in a real mobile robot. One strategy of nonlinear control gaining popularity among researchers is the differential-flatness-based control (Veslin Diaz et al., 2011; Levine, 2009). It has been investigated to control flexible robots (Markus et al., 2012; Markus et al., 2017), mobile robots (Coulaud and Campion, 2007), under-actuated planar robots (Vivek et al., 2010), and so on. Differential flatness is known to be well suited for the problem of trajectory generation and tracking (Markus et al., 2013). With

Nasr, S., Abadi, A., Bouallegue, K. and Mekki, H.
Control of the Chaotic Phenomenon in Robot Path using Differential Flatness.
DOI: 10.5220/0006828302370243
In *Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2018) - Volume 2*, pages 237-243
ISBN: 978-989-758-321-6

this strategy, the trajectories (position, velocity and acceleration) of a nonlinear system can be easily interpolated by defining a smooth curve with initial and final conditions. The control variables and state can be reconstructed without having to integrate the system equations(Levine, 2009). Thus, we utilize the flatness control method to solve the problem of path planning and chaotic phenomena , which can appear in the robot trajectory; and we ensure that the mobile robot tracks this trajectory.

This paper is organized as follows. In section 2, we explain the basic description model of the robot. The basic definition and control strategy of the differential flatness theory is presented in section 3. We describe the kinematic system and its flatness property and we propose the control law to solve the trajectory tracking problem. In section 4, we present the chaotic phenomenon and the control law to solve the trajectory problem. We give the concluding remarks in section 5.

2 MODEL DESCRIPTION

The mobile robot considered in this work is a differential motion robot with two degrees of freedom, composed by two independent active wheels, and a third passive wheel (a kind of standard free-wheel). This type of robots represents an important compromise between the simplicity of control and the degrees of freedom that allow the robot to accomplish the mobility requirements (Siegwart et al., 2011).

The robot structure is considered as a rigid body operating on the horizontal plane (figure 1).Its kinematic model can be described as a differential system composed of two control parameters, v and ω, which respectively represent the values of linear and angular speeds. The state equation of the wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v(t) \\ \omega(t) \end{pmatrix} \quad (1)$$

where x and y are the position of the robot and θ is the orientation angle of the robot. The robot displacement control can be performed by supplying the linear and angular velocities of the body, $v(t)$ and $\omega(t)$, called control variables or inputs.

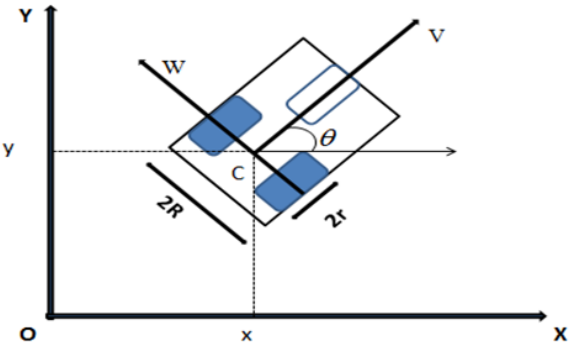

Figure 1: Geometry of mobile robot on Cartesian plane.

3 FLATNESS CONTROL METHOD

Flatness is a characteristic or property of a particular system in which all solutions of the system can be parameterized by a finite number of functions and their derivatives (Nicolau and Respondek, 2013). For the analysis and design of controllers for nonlinear systems with this characteristic, this mathematical property is extensively used.

3.1 Flatness Theory

Differential flatness is a property of control systems Dynamics, as presented by Fliess et al. (Fliess et al., 1995). Differential flatness, provides a unified analysis framework for trajectory planning and control of nonlinear systems. This is particularly useful for non-linear sub-actuated systems where it is difficult to plan and to analytically design possible trajectories. The necessary condition for a control system to be differentially flat is that it must be controlled.

From a control perspective, a good explanation of differential flatness for any nonlinear systems of the form,

$$\dot{x} = f(x,u); x \in R^n, u \in R^m \quad (2)$$

The system can be stated to be differentially flat if and only if there exists a finite set of independent variables, equal to the number of inputs, called flat outputs $y = [y_1, \ldots, y_m]^T$ in such a way that :

$$y = y\left(x, u, \dot{u}, \ldots, u^{(p)}\right) \quad (3)$$

$$x = x(y, \dot{y}, \ddot{y}, \ldots, y^{(r)}) \quad (4)$$

$$u = u(y, \dot{y}, \ddot{y}, \ldots, y^{(q)}) \quad (5)$$

Moreover, for a flat system, there is an invertible input and state transformations that can transform non-linear systems into linear canonical forms (controllable linear chain of integrators). An arbitrary trajectory for flat outputs corresponds to the original state of the system of reference trajectories.This makes planning possible in the flat output domain. In addition, the linear feedback of the control can be designed in the field of linear flat outputs by closing the loop on errors in the flat outputs and their derivatives.

3.2 Flatness Control Strategy

The Control design and trajectory planning for flat systems are relatively easy because the trajectory can be defined in terms of flat outputs while the required control input can be obtained using the flatness property.

In order to prove how the kinematic model of the mobile robot is differentially flat, we choose the Cartesian position of the robot center (x,y) as flat outputs. To design a diffeomorphism between flat outputs and their derivatives and original states, the input prolongation is utilized. Prolongation is a crucial method used where the vector representing the state is extended by some system parameters which is used to describe a particular system as a differentially flat system. A very common prolongation way means is the input prolongation where the input also becomes a state. This property is utilized in optimal trajectory generation and tracking control laws.

Now, on performing one prolongation of v as an additional state, we describe the prolonged systems by:

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{v} = \bar{U}_1 \\ \dot{\theta} = \bar{U}_2 \end{cases} \quad (6)$$

Here, $\bar{U}_1$, $\bar{U}_2$ are the new inputs for the prolonged system that satisfy:

$$\begin{array}{l} \bar{U}_1 = \dot{v} \\ \bar{U}_2 = \omega \end{array} \quad (7)$$

By choosing the flat outputs

$$Fo = [Fo_1, Fo_2]^T = [x,y]^T \quad (8)$$

All the inputs and the state variables can be expressed in terms of flat outputs and their derivatives. With $(x,y) = (Fo_1, Fo_2)$

$$v = \sqrt{\dot{F}o_1{}^2 + \dot{F}o_2{}^2}, \theta = \arctan\left(\frac{\dot{F}o_2}{\dot{F}o_1}\right), \quad (9)$$

The inputs $\bar{U}_1, \bar{U}_2$ can be defined as follows:

$$\bar{U}_1 = \dot{v} = \frac{\dot{F}o_1\ddot{F}o_1 + \dot{F}o_2\ddot{F}o_2}{\sqrt{\dot{F}o_1{}^2 + \dot{F}o_2{}^2}} \quad (10)$$

$$\bar{U}_2 = \dot{\theta} = \frac{\dot{F}o_1\ddot{F}o_2 + \ddot{F}o_1\dot{F}o_2}{\dot{F}o_1{}^2 + \dot{F}o_2{}^2} \quad (11)$$

By differentiating the flat outputs up to an input appears, an invertible relation between inputs and higher derivatives of the flat outputs can be equivalently build from equation 10 and equation 11 as described follows:

$$\begin{pmatrix} \ddot{F}o_1 \\ \ddot{F}o_2 \end{pmatrix} = D \begin{pmatrix} \bar{U}_1 \\ \bar{U}_2 \end{pmatrix} \quad (12)$$

With

$$D = \begin{pmatrix} \cos\theta & -v\sin\theta \\ \sin\theta & v\cos\theta \end{pmatrix} \quad (13)$$

the inputs are choosing as

$$\begin{pmatrix} \bar{U}_1 \\ \bar{U}_2 \end{pmatrix} = D^{-1}V = \frac{1}{v}\begin{pmatrix} v\cos\theta & v\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} V \quad (14)$$

Then equation 12 can be written as

$$\ddot{F}o = V. \quad (15)$$

The reference trajectory must allow the robot to move, from an initial position with coordinates (x,y) at time t=0 to a final position with coordinates (xf, yf) at time t = 10 s, with minimum of energy and also avoid some static circular obstacles. These obstacles are defined by the following equation:

$$Ob_i = (x - x_r)^2 + (y - y_r)^2 - R \quad (16)$$

Where $x_r and y_r$ are the coordinates of the center of the circle and r denotes the radius, i is the number of obstacles.

The constraint which means that the mobile robot avoids the obstacle is defined as follows:

$$Ob_1(x,y) = (x-2)^2 + (y-2)^2 - 1 \geq 0 \quad (27)$$

$$Ob_2(x,y) = (x-6)^2 + (y-3)^2 - 1 \geq 0 \quad (28)$$

$$Ob_3(x,y) = (x-8)^2 + (y-5)^2 - 1 \geq 0 \quad (29)$$

$$Ob_4(x,y) = (x-6)^2 + (y-6)^2 - 1 \geq 0 \quad (29)$$

$$Ob_5(x,y) = (x-2)^2 + (y-6)^2 - 1 \geq 0 \quad (29)$$

To meet these objectives, the problem of reference trajectory generation is formulated as an optimization problem in the following way:

$$\min \sqrt{\dot{x}^2 + \dot{y}^2} \tag{17}$$

$$Ob_i \geq 0 \tag{18}$$

This problem of optimization is solved by the most efficient method based on the flatness and the B-spline function (Bahrami et al., 2009).

$$\begin{array}{ll} x(0) = 0 & x(10) = 9 \\ y(0) = 0 & x(10) = 9 \\ \theta(0) = 0 & \theta(10) = 0 \\ v(0) = 0 & v(10) = 0 \end{array} \tag{19}$$

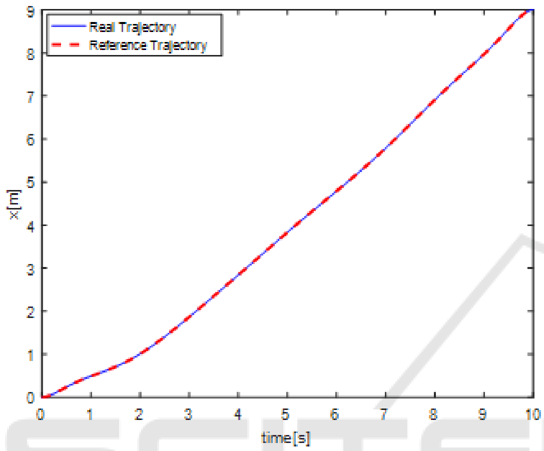

Figure 2: Simulation results of reference and real trajectories of x position.

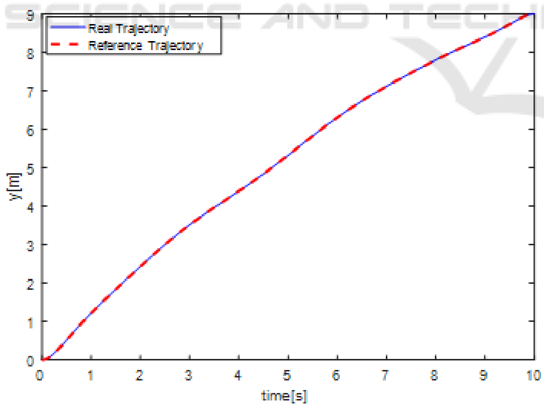

Figure 3: Simulation results of reference and the real trajectories of y position.

In Figure 2and 3, we show that the flatness control input defined by equation 10 and equation 11 permits a good tracking of the desired trajectory for the mobile Robot. Therefore, the flatness property is considered as a powerful tool for path planning and tracking trajectory. As depicted in Figure6, the mobile robot can easily avoid the defined static obstacle.

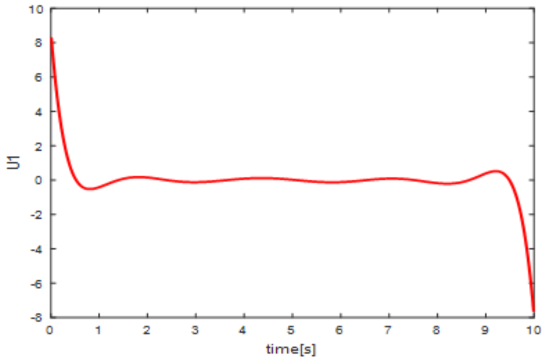

Figure 4: Simulation results of the control input $U1$.

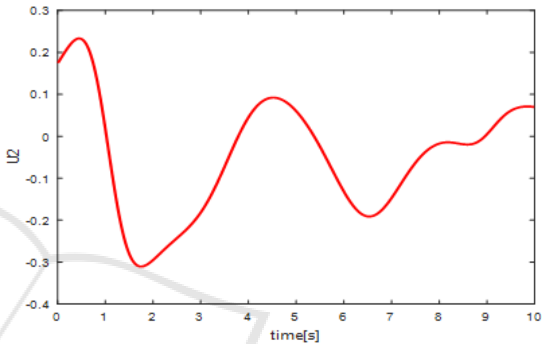

Figure 5: Simulation results of the control input $U2$.

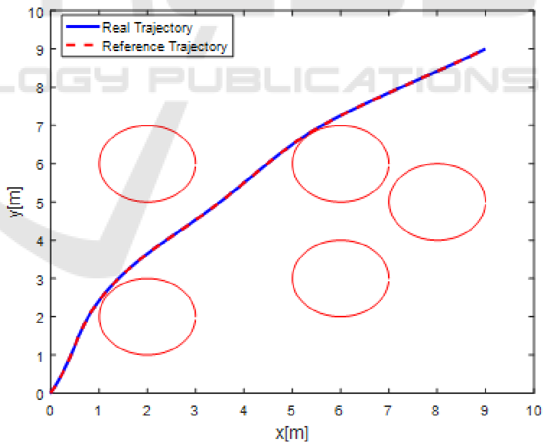

Figure 6: Simulation results of optimal trajectory with obstacle avoidance.

4 CHAOTIC PHENOMENA

Deterministic chaos has been employed for developing consumer electronic products and intelligent industrial systems.

4.1 Chaos Theory

During the 20th century, three great revolutions occurred: quantum mechanics, relativity and chaos. The theory of chaos, also called the dynamical system theory, is the study of unstable aperiodic behavior in deterministic dynamical systems, which show a sensitive dependence on initial conditions (Vaidyanathan, 2013).

The chaos theory has drawn a great deal of attention in the scientific community for almost two decades. Chaos is a very interesting phenomenon in nonlinear dynamical systems, which has been intensively studied during the last decades and used in several possible commercial applications (Trejo-Guerra, 2008).

The Lorenz system has become one of paradigms in the research of chaotic systems. The Lorenz chaotic system is utilized for investigation. The dynamical equations of the Lorenz system is given as follows:

$$\begin{cases} \dot{X}_1 = -10X_1 + 10.X_2 \\ \dot{X}_2 = 28X_1 - X_2 - X_1.X_3 \\ \dot{X}_3 = -\frac{8}{3}X_3 + X_1.X_2 \end{cases} \quad (20)$$

The implementation of this dynamic system is presented in figure 7.

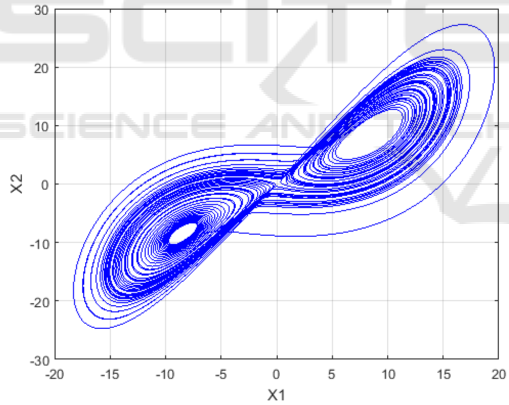

Figure 7: Lorenz chaotic system.

4.2 Chaos Analysis in Mobile Robot

The most applications of chaos in robotics are classified into two types: chaos synthesis and chaos analysis ; chaos synthesis in robotics is defined as the application of chaotic systems for motion planning of autonomous robots and entails the generation of artificial chaos to make different mobile robots accomplish specific tasks (Aihara and Katayama, 1995). Whereas, chaos analysis implies the observation of chaotic behavior in autonomous robots. Therefore, controlling the chaotic behavior of the mobile robot becomes a worthwhile endeavor.

In this subsection, based on the Lorenz chaotic system, we give a chaotic behavior to the mobile robot. Subsequently, we use the control technique based on differential flatness to control this chaotic behavior in order to allow the robot to complete its trajectory, despite its behavior, and to achieve its objective.

By using the dynamic equation of the Lorenz system, introduced in equation 20, we will find the robot equation of motion as follows:

$$\begin{cases} \dot{X}_1 = -10X_1 + 10.X_2 \\ \dot{X}_2 = 28X_1 - X_2 - X_1.X_3 \\ \dot{X}_3 = -\frac{8}{3}X_3 + X_1.X_2 \\ \dot{x} = v\cos(X_1) \\ \dot{y} = v\sin(X_1) \end{cases} \quad (21)$$

The proposed system described in equation 21 generates an unpredictable path by giving a chaotic behavior of the mobile robot with two independent active wheels.

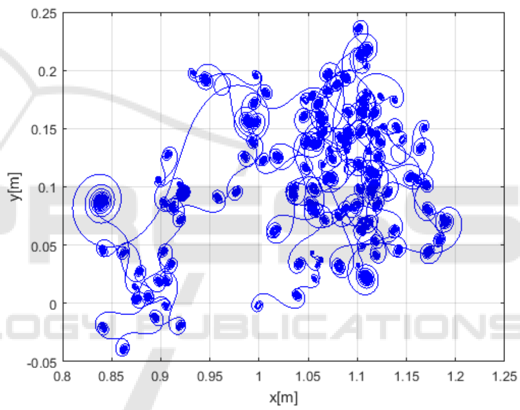

Figure 8: Chaotic phenomena in mobile robot.

As depicted in figure 8, the sensitivity to initial conditions makes the robot trajectory extremely unpredictable. Then with this behavior, the robot can not reach its objective. Thus, moving from an initial position to a final one is almost impossible with this behavior.

In this context, control over flatness may be a good solution to solve this problem. We adopt the technique used in section 3 to restore the control of the new kinematic system combined with the Lorenz chaotic system. In this case, we choose $\theta = X_1$.

5 DISCUSSION

Figures 2, 3, 4 and 5 illustrate the effectiveness of the closed-loop flatness control which allows the mobile robot to follow the desired reference trajectory properly. By ensuring a good tracking of the trajectory,

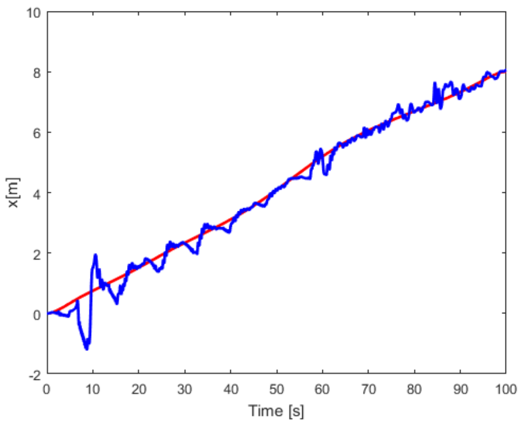

Figure 9: Flatness control of x chaotic trajectory of mobile robot.

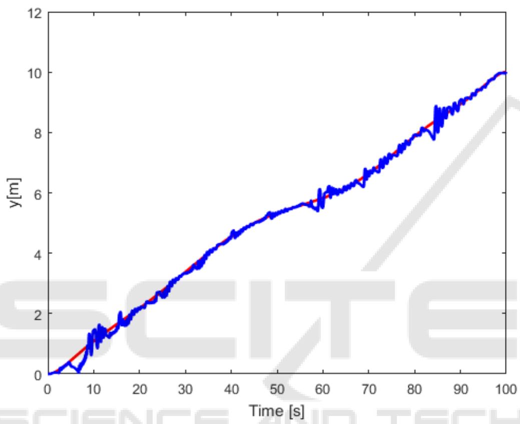

Figure 10: Flatness control of y chaotic trajectory of mobile robot.

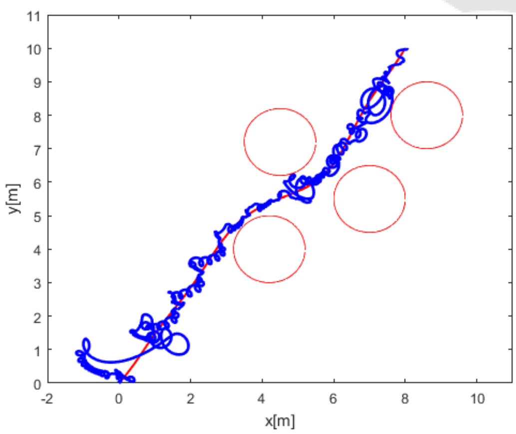

Figure 11: Flatness control of x-y chaotic trajectory of mobile robot with obstacle avoidance.

the mobile robot can move by avoiding the static obstacles with the minimal energy and by choosing the optimal trajectory.

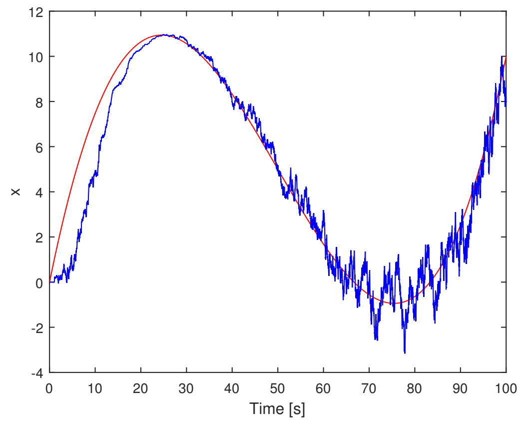

Figure 12: Flatness control of second chaotic trajectory of mobile robot.

Figure 7 shows the behavior of the Lorenz chaotic system in the Cartesian plan.

In figures 9, 10 and 12 we present the good effectiveness of chaotic trajectory tracking , and we ensure that the robot better reaches its desired trajectory. Even more, as depicted in figure 11, we can show the robustness of the control strategy with chaotic phenomenon and in the presence of obstacles.

Next, some endeavors for uncovering the chaotic behavior of robots are presented. Chaos can be employed for analyzing robotic arms, and chaos quantifiers can be used for analyzing chaotic dynamics in robot-environment interaction.

6 CONCLUSIONS

This article has described the path planning and the flatness based tracking control of a wheeled mobile robot. The flatness-based approach to trajectory control and optimal trajectory tracking offers a fast alternative to the classical control for such robots. Having determined the flat output of the mobile robot, the trajectory control has been determined with reasonable accuracy. Secondly, we have presented a chaotic phenomenon tuned to the behavior of the autonomous mobile robot, so we have solved the problem related to this phenomenon using the differential flatness method.

In recent years, the discovery of chaos has attracted much interest among investigators. Deterministic chaos leads to a quantitative analysis, which is the essence of science. In spite of several efforts to find evidence of chaotic dynamics in robotics, useful applications of deterministic chaos in robotics have rarely been studied.

REFERENCES

Aihara, K. and Katayama, R. (1995). Chaos engineering in japan. *Communications of the ACM*, 38(11):103–107.

Bahrami, M., Jamilnia, R., and Naghash, A. (2009). Trajectory optimization of space manipulators with flexible links using a new approach. *International Journal of Robotics*, 1(1):48–55.

Bakdi, A., Hentout, A., Boutami, H., Maoudj, A., Hachour, O., and Bouzouia, B. (2017). Optimal path planning and execution for mobile robots using genetic algorithm and adaptive fuzzy-logic control. *Robotics and Autonomous Systems*, 89:95–109.

Belaidi, H., Bentarzi, H., and Belaidi, M. (2017). Implementation of a mobile robot platform navigating in dynamic environment. In *MATEC Web of Conferences*, volume 95, page 08004. EDP Sciences.

Coulaud, J. and Campion, G. (2007). Optimal trajectory tracking for differentially flat systems with singularities. In *Control and Automation, 2007. ICCA 2007. IEEE International Conference on*, pages 1960–1965. IEEE.

Fliess, M., Lévine, J., Martin, P., and Rouchon, P. (1995). Flatness and defect of non-linear systems: introductory theory and examples. *International journal of control*, 61(6):1327–1361.

Hargas, Y., Mokrane, A., Hentout, A., Hachour, O., and Bouzouia, B. (2015). Mobile manipulator path planning based on artificial potential field: Application on robuter/ulm. In *Electrical Engineering (ICEE), 2015 4th International Conference on*, pages 1–6. IEEE.

Jansri, A., Klomkarn, K., and Sooraksa, P. (2004). On comparison of attractors for chaotic mobile robots. In *Industrial Electronics Society, 2004. IECON 2004. 30th Annual Conference of IEEE*, volume 3, pages 2536–2541. IEEE.

Klančar, G. and Škrjanc, I. (2007). Tracking-error model-based predictive control for mobile robots in real time. *Robotics and Autonomous Systems*, 55(6):460–469.

Korayem, M., Yousefzadeh, M., and Manteghi, S. (2016). Dynamics and input–output feedback linearization control of a wheeled mobile cable-driven parallel robot. *Multibody System Dynamics*, pages 1–19.

Kumar, N., Panwar, V., Borm, J.-H., and Chai, J. (2014). Enhancing precision performance of trajectory tracking controller for robot manipulators using rbfnn and adaptive bound. *Applied Mathematics and Computation*, 231:320–328.

Lai, C. Y. (2014). Improving the transient performance in robotics force control using nonlinear damping. In *Advanced Intelligent Mechatronics (AIM), 2014 IEEE/ASME International Conference on*, pages 892–897. IEEE.

Levine, J. (2009). *Analysis and control of nonlinear systems: A flatness-based approach*. Springer Science & Business Media.

Markus, E., Agee, J., Jimoh, A., Tlale, N., and Zafer, B. (2012). Flatness based control of a 2 dof single link flexible joint manipulator. In *SIMULTECH*, pages 437–442.

Markus, E. D., Agee, J. T., and Jimoh, A. A. (2013). Trajectory control of a two-link robot manipulator in the presence of gravity and friction. In *AFRICON, 2013*, pages 1–5. IEEE.

Markus, E. D., Agee, J. T., and Jimoh, A. A. (2017). Flat control of industrial robotic manipulators. *Robotics and Autonomous Systems*, 87:226–236.

Martins-Filho, L. S., Machado, R. F., Rocha, R., and Vale, V. (2004). Commanding mobile robots with chaos. In *ABCM Symposium Series in Mechatronics*, volume 1, pages 40–46.

Nakamura, Y. and Sekiguchi, A. (2001). The chaotic mobile robot. *IEEE Transactions on Robotics and Automation*, 17(6):898–904.

Nehmzow, U. (2003). Quantitative analysis of robot–environment interactiontowards scientific mobile robotics. *Robotics and Autonomous Systems*, 44(1):55–68.

Nicolau, F. and Respondek, W. (2013). Multi-input control-affine systems linearizable via one-fold prolongation and their flatness. In *Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on*, pages 3249–3254. IEEE.

Siegwart, R., Nourbakhsh, I. R., and Scaramuzza, D. (2011). *Introduction to autonomous mobile robots*. MIT press.

Tinh, N. V., Linh, N. T., Cat, P. T., Tuan, P. M., Anh, M. N., and Anh, N. P. (2016). Modeling and feedback linearization control of a nonholonomic wheeled mobile robot with longitudinal, lateral slips. In *Automation Science and Engineering (CASE), 2016 IEEE International Conference on*, pages 996–1001. IEEE.

Trejo-Guerra, R., T.-C. E. C.-H. C. S.-L. C. F. M. (2008). Current conveyor realization of synchronized chuas circuits for binary communications. *IEEE . DTIS*, pages 1–4.

Vaidyanathan, S. (2013). Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters. *Journal of Engineering Science and Technology Review*, 6(4):53–65.

Veslin Diaz, E., Slama, J., Dutra, M., Lengerke, O., and Morales Tavera, M. (2011). Trajectory tracking for robot manipulators using differential flatness. *Ingeniería e Investigación*, 31(2):84–90.

Vivek, S., Sunil, K., Jaume, F., et al. (2010). Differential flatness of a class of n-dof planar manipulators driven by 1 or 2 actuators.