A flatness controller for a mobile robot in presence of the chaotic phenomena

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Abstract: In this paper, based on differential flatness theory, the motion control of a wheeled mobile robot is studied. However, a flatness-based controller is designed to ensure the trajectory tracking. Secondly, this paper deal about the complex chaotic behaviors which can appear in the dynamic trajectory of an mobile robot. Different mathematical tools have been used such as flatness control technique and non linear chaotic system. Simulation results for kinematic controller is presented to demonstrate the effectiveness of this approach.

Key–Words: Mobile robot, flatness control, chaos control, chaotic phenomena.

1 Introduction

The control of mobile robots has been the subject of much research in recent years, due to increasingly frequent use in dangerous or inaccessible environments where human beings can hardly intervene. For autonomous mobile robotics, path generation and execution is one of the most important tasks. Path planning is the process of generating a sequence of trajectory deriving from the assigned task to the mobile robot to be able to perform it.

The general problem is reduced in most cases to move the robot in a known or unknown environment [1], while avoiding any fixed or mobile obstacles, to carry out a prescribed task. It follows that it is necessary to be able to define a strategy of movement (path planning) [2], then to execute the prescribed displacement.

The robot controller, which is a major component, has received a lot of attention from researchers. This is why it has a direct impact on their robustness and could prevent their deployment and applicability in several domains [3, 4]. Many control techniques have been proposed for modern robots including the classical PID, feedback linearization [5, 6], inverse dynamics, model predictive control [7], adaptive fuzzy-logic control [8] etc.

Up to now, there has been no experimental work that has treated the chaotic phenomena in the robot trajector.y On the other hand, the interaction between the theory of chaos and mobile robotics has been only recently studied, as can be seen in [9], for the generation of the unpredictable trajectory for the robot. For example, integration between a chaotic system and the robot's motion system, dynamic systems, is used to impart chaotic behavior to a robot like the Arnold system in [10]. An extension of this strategy, applying various chaotic systems on integration with the kinematics model of robot, can be found in [11]. In [12], the author proposed an open loop control approach to produce unpredictable trajectories so that to control the velocities of the robot's wheels the state variables of the Lorenz chaotic system are used.But there has been no research work to solve the chaotic phenomena problem that can appear in the robot trajectory.

In this context we propose to use a controller to solve this problem and to facilitate the implementation of our work in a real mobile robot. One strategy of nonlinear control gaining popularity among researchers is the differential flatness based control [13, 14]. It has been investigated to control the flexible robots [15, 16], the mobile robots [17], the underactuated planar robots [18], and so on. Differential flatness is known to be well suited for the problem of trajectory generation and tracking [19]. With this strategy, the trajectories (position, velocity and acceleration) of a nonlinear system can be easily interpolated by defining a smooth curve with initial and final conditions. The control variables and state can be reconstructed without having to integrate the system equations [14]. Thus, we utilize the flatness control method to solve the problem of path planning and the chaotic phenomena , which can appear in the robot trajectory, and we ensure that the mobile robot tracks this trajectory.

This paper is organized as follows. In Section 2, we have explained the basic description model of robot. Basic definition and control strategy of differential flatness theory is presented in section 3. We have described the kinematic system and its flatness property and we have proposed the control law to solve the trajectory tracking problem. In Section 4, we have presented the chaotic phenomena and the control law to solve the trajectory problem. We have given the concluding remarks in section 5.

2 Model description

The mobile robot considered in this work is a differential motion robot with two degrees of freedom, composed by two independent active wheels, and a third passive wheel (a kind of standard free-wheel). This type of robot represents an important compromise between the simplicity of the control and the degrees of freedom that allow the robot to accomplish the mobility requirements [20].

The robot structure is considered as a rigid body operating on the horizontal plane(figure 1).Its kinematic model can be described as a differential system comprising of two control parameters v and ω which represent respectively the values of linear and angular speeds. The state equation of the wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v(t) \\ \omega(t) \end{pmatrix} \quad (1)$$

where x and y are the position of the robot and θ is the orientation angle of the robot. The robot displacement control can be performed by supplying the linear and angular velocities of the body, $v(t)$ and $\omega(t)$, called control variables or input.

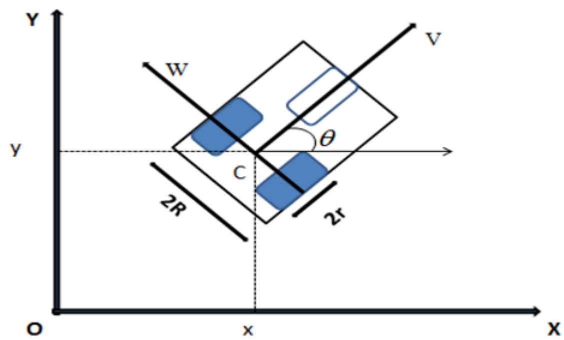

Figure 1: Geometry of the mobile robot on the Cartesian plane

3 Flatness control method

Flatness is a characteristic or property of a particular system in which all solutions of the system can be parameterized by a finite number of functions and their derivatives [21]. For the analysis and design of controllers for nonlinear systems with this character, this mathematical property is extensively used.

3.1 Flatness theory

Differential flatness is a property of control systems Dynamics, Fliess et al. [22]. Differential flatness, provides a unified analysis framework for trajectory planning and control of nonlinear systems. This is particularly useful for non-linear sub-actuated systems where it is difficult to plan and analytically design possible trajectories. The necessary condition for a control system to be differentially flat is that it must be controlled.

From a control perspective, a good explanation of differential flatness for any nonlinear systems of the form,

$$\dot{x} = f\,(x, u)\,; x \in R^n, u \in R^m \quad (2)$$

the system can be stated to be differentially flat if and only if there exists a finite set of independent variables, equal to the number of inputs, called flat outputs $y = [y_1, ..., y_m]^T$ in such a way that :

$$y = y\left(x, u, \dot{u}, ..., u^{(p)}\right). \quad (3)$$

$$x = x(y, \dot{y}, \ddot{y}, ..., y^{(r)}) \quad (4)$$

$$u = u(y, \dot{y}, \ddot{y}, ..., y^{(q)}) \quad (5)$$

Moreover, for a flat system, there is an invertible input and state transformations that can transform non-linear systems into linear canonical forms (controllable linear chain of integrators). An arbitrary trajectory for flat outputs corresponds to the original state of the system of reference trajectories.This makes planning possible in the flat output domain. In addition, the linear feedback of the control can be designed in the field of linear flat outputs by closing the loop on errors in the flat outputs and their derivatives.

3.2 Flatness control strategy

The Control design and trajectory planning for flat systems are relatively easy because the trajectory can be defined in terms of flat outputs while the required control input can be obtained using the flatness property.

In order to prove how the kinematic model of the mobile robot is differentially flat, we choose the Cartesian position of the robot center (x, y) as the flat outputs. To design a diffeomorphism between flat outputs and their derivatives and original states, the input prolongation is utilized. Prolongation is a crucial method used where the vector representing the state is extended by some system parameters which is used to describe a particular system as a differentially flat system. A very common prolongation way means is the input prolongation where the input also becomes a state. This property is utilized in optimal trajectory generation and tracking control laws.

Now, on performing one prolongation of v as an additional state, we describe the prolonged systems by:

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{v} = \bar{U}_1 \\ \dot{\theta} = \bar{U}_2 \end{cases} \tag{6}$$

Here, $\bar{U}_1, \bar{U}_2$ are the new inputs for the prolonged system that satisfy:

$$\begin{array}{l} \bar{U}_1 = \dot{v} \\ \bar{U}_2 = \omega \end{array} \tag{7}$$

By choosing the flat outputs

$$Fo = [Fo_1, Fo_2]^T = [x, y]^T \tag{8}$$

All the inputs and the state variables can be expressed in terms of flat outputs and their derivatives. With $(x, y) = (Fo_1, Fo_2)$

$$v = \sqrt{\dot{Fo_1}^2 + \dot{Fo_2}^2}, \theta = \arctan\left(\frac{\dot{Fo_2}}{\dot{Fo_1}}\right), \tag{9}$$

The inputs $\bar{U}_1, \bar{U}_2$ can be defined as follows:

$$\bar{U}_1 = \dot{v} = \frac{\dot{Fo_1}\ddot{Fo_1} + \dot{Fo_2}\ddot{Fo_2}}{\sqrt{\dot{Fo_1}^2 + \dot{Fo_2}^2}} \tag{10}$$

$$\bar{U}_2 = \dot{\theta} = \frac{\dot{Fo_1}\ddot{Fo_2} + \ddot{Fo_1}\dot{Fo_2}}{\dot{Fo_1}^2 + \dot{Fo_2}^2} \tag{11}$$

By differentiating the flat outputs up to an input appears, an invertible relation between inputs and higher derivatives of the flat outputs can be equivalently build from equation 10 and equation 11 as described follows:

$$\begin{pmatrix} \ddot{Fo_1} \\ \ddot{Fo_2} \end{pmatrix} = D \begin{pmatrix} \bar{U}_1 \\ \bar{U}_2 \end{pmatrix} \tag{12}$$

Where

$$D = \begin{pmatrix} \cos\theta & -v\sin\theta \\ \sin\theta & v\cos\theta \end{pmatrix} \tag{13}$$

the inputs are choosing as

$$\begin{pmatrix} \bar{U}_1 \\ \bar{U}_2 \end{pmatrix} = D^{-1}V = \frac{1}{v}\begin{pmatrix} v\cos\theta & v\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} V \tag{14}$$

Then equation 12 can be written as

$$\ddot{Fo} = V. \tag{15}$$

The reference trajectory must allow the robot to move, from an initial position with coordinates (x, y) at time t=0 to a final position with coordinates (xf, yf) at time t = 10 s, with minimum of energy and also avoid some static circular obstacles. These obstacles are defined by the following equation:

$$Ob_i = (x - x_r)^2 + (y - y_r)^2 - R \tag{16}$$

Where $x_r and y_r$ are the coordinates of the center of the circle and r denotes the radius, i is the number of obstacles.

The constraint which means that the mobile robot avoids the obstacle is defined as follows:

$$Ob_1(x, y) = (x-2)^2 + (y-2)^2 - 1 \geq 0 \tag{27}$$

$$Ob_2(x, y) = (x-6)^2 + (y-3)^2 - 1 \geq 0 \tag{28}$$

$$Ob_3(x, y) = (x-8)^2 + (y-5)^2 - 1 \geq 0 \tag{29}$$

$$Ob_4(x, y) = (x-6)^2 + (y-6)^2 - 1 \geq 0 \tag{29}$$

$$Ob_5(x, y) = (x-2)^2 + (y-6)^2 - 1 \geq 0 \tag{29}$$

To meet these objectives, the problem of reference trajectory generation is formulated as an optimization problem in the following way:

$$\min \sqrt{\dot{x}^2 + \dot{y}^2} \tag{17}$$

$$Ob_i \geq 0 \tag{18}$$

This problem of optimization is solved by the most efficient method based on the flatness and the B-spline function [23].

$$\begin{array}{ll} x(0) = 0 & x(10) = 9 \\ y(0) = 0 & x(10) = 9 \\ \theta(0) = 0 & \theta(10) = 0 \\ v(0) = 0 & v(10) = 0 \end{array} \tag{19}$$

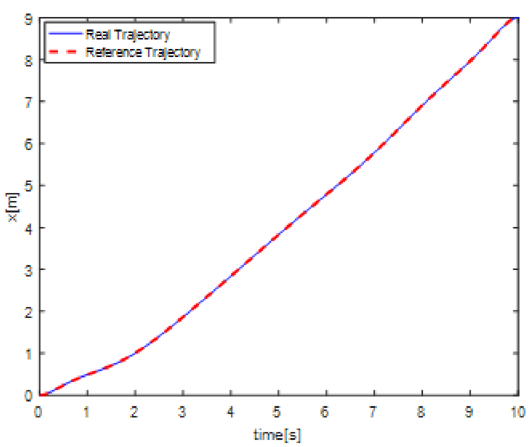

Figure 2: Simulation results of reference and real trajectories of x position

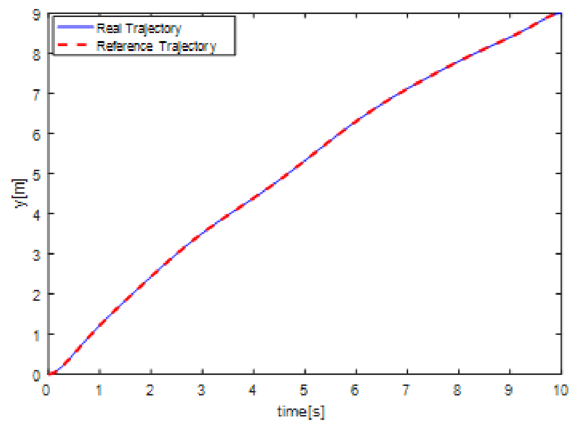

Figure 3: Simulation results of reference and the real trajectories of y position

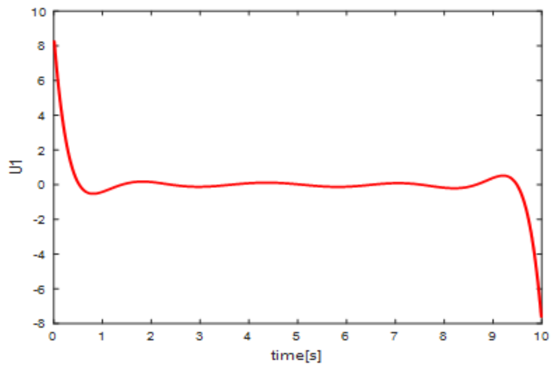

Figure 4: Simulation results of the control input $U1$

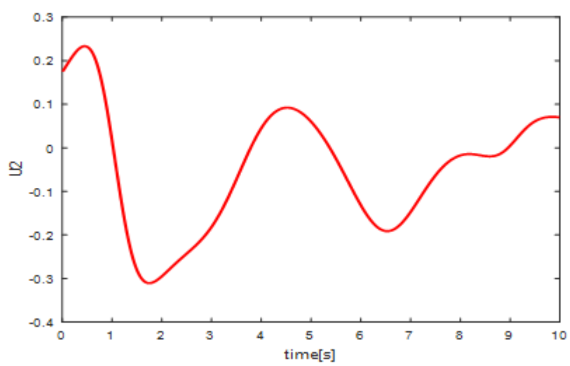

Figure 5: Simulation results of the control input $U2$

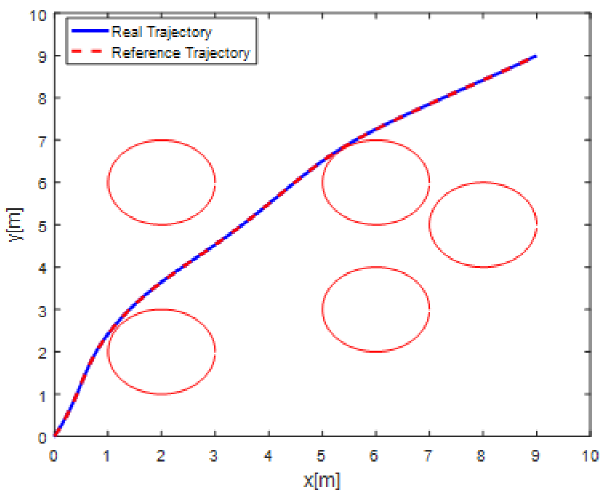

Figure 6: Simulation results of optimal trajectory with obstacle avoidance

In Figure 2and 3, we show that the flatness control input defined by equation 10 and equation 11 permits a good tracking of the desired trajectory for the mobile Robot. Therefore, the flatness property is considered as a powerful tool for path planning and tracking trajectory. As depicted in Figure6, the mobile robot can easily avoid the defined static obstacle.

4 Chaotic phenomena

Deterministic chaos has been employed for developing consumer electronic products and intelligent industrial systems.

4.1 Chaos theory

During the 20th century, three great revolutions occurred: quantum mechanics, relativity and chaos. The theory of chaos, also called dynamical systems theory, is the study of unstable aperiodic behavior in deterministic dynamical systems, which show a sensitive dependence on initial conditions [24].

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Chaos is a very interesting phenomenon in nonlinear dynamical systems, which has been intensively studied during the last decades and used in several possible commercial applications [25].

The Lorenz system has become one of paradigms in the research of chaotic systems. Lorenz chaotic system is utilized for the investigation. The dynamical equations of Lorenz system is given as follows:

$$\begin{cases} \dot{X}_1 = -10X_1 + 10.X_2 \\ \dot{X}_2 = 28X_1 - X_2 - X_1.X_3 \\ \dot{X}_3 = -\frac{8}{3}X_3 + X_1.X_2 \end{cases} \quad (20)$$

The implementation of this dynamic system is presented in figure 7.

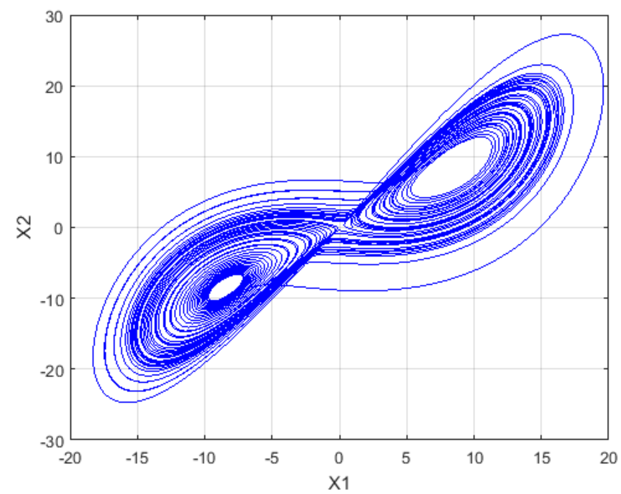

Figure 7: Lorenz chaotic system

4.2 Chaos analysis in mobile robot

The most applications of chaos in robotics are classified into two types: chaos synthesis and chaos analysis ; chaos synthesis in robotics is defined as the application of chaotic systems for motion planning of autonomous robots and entails the generation of artificial chaos to make different mobile robots accomplish specific tasks [26], whereas chaos analysis implies the observation of chaotic behaviour in autonomous robots. Therefore, controlling the chaotic behavior of the mobile robot becomes a worthwhile endeavor.

In this subsection, based on Lorenz's chaotic system, we give chaotic behavior to the mobile robot. Subsequently, we use the control technique based on differential flatness to control this chaotic behavior in order to allow the robot to complete its trajectory, despite its behavior, and to achieve its objective.

By using the dynamic equation of Lorenz system introduced in equation 20, we will find robot equation of motion as follows:

$$\begin{cases} \dot{X}_1 = -10X_1 + 10.X_2 \\ \dot{X}_2 = 28X_1 - X_2 - X_1.X_3 \\ \dot{X}_3 = -\frac{8}{3}X_3 + X_1.X_2 \\ \dot{x} = v\cos(X_1) \\ \dot{y} = v\sin(X_1) \end{cases} \quad (21)$$

The proposed system described in equation 21 generates an unpredictable path by giving a chaotic behavior of the mobile robot with two independent active wheels.

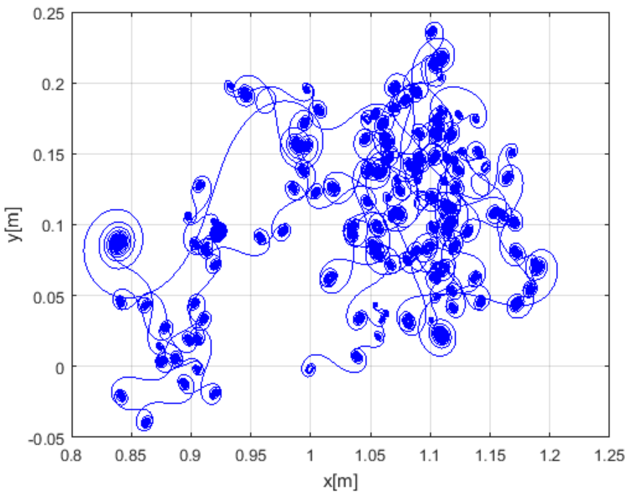

Figure 8: Chaotic phenomena for mobile robot

As depicted in figure 8, the sensitivity to initial conditions makes the trajectory of robot extremely unpredictable. Then with this behavior the robot can not reach its objective. So moving from an initial position to an final position is almost impossible with this behavior.

In this context, control over flatness may be a good solution to solve this problem. We adopt the technique used in Section 3 to restore the control of the new kinematic system combined with the Lorenz chaotic system. In this case, we choose $\theta = X_1$

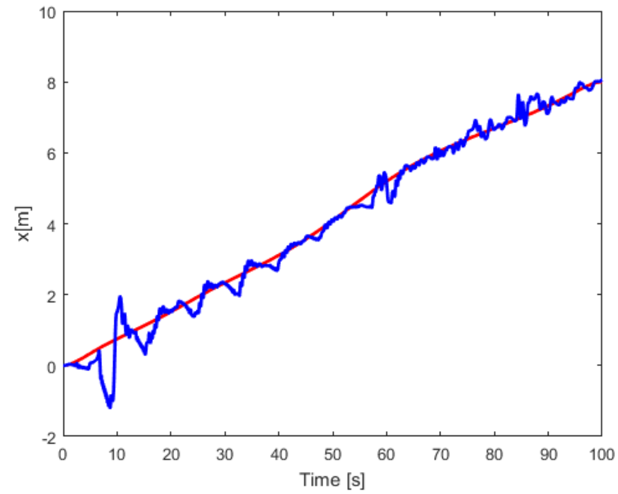

Figure 9: Flatness control of x chaotic trajectory of the mobile robot

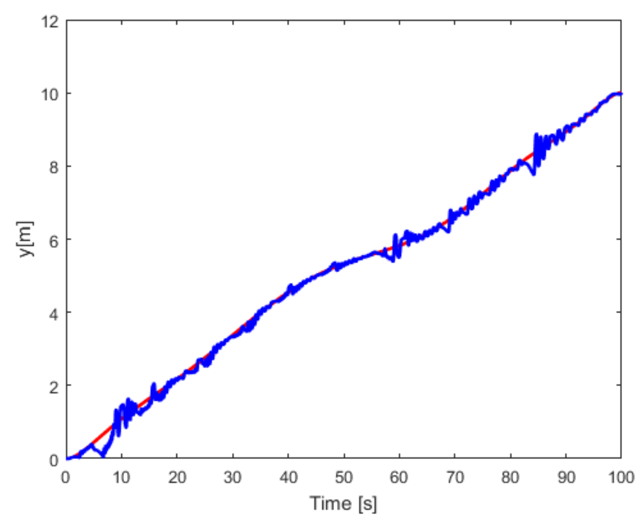

Figure 10: Flatness control of y chaotic trajectory of the mobile robot

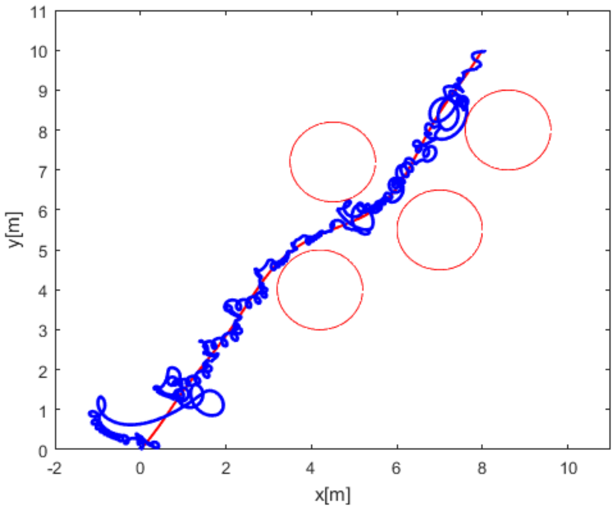

Figure 11: Flatness control of x-y chaotic trajectory of the mobile robot with obstacle avoidance

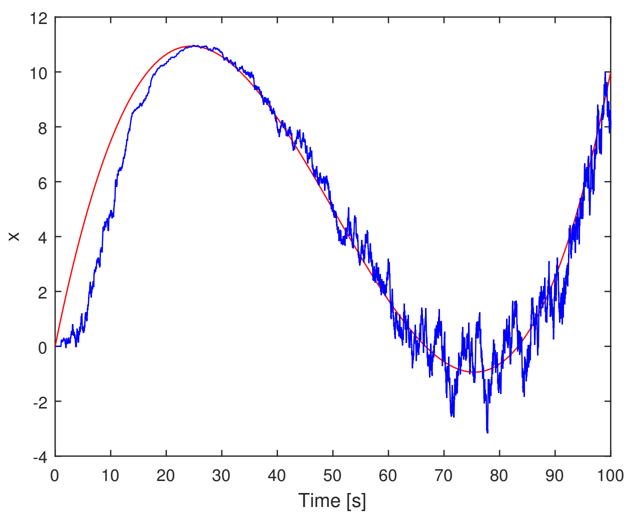

Figure 12: Flatness control of second chaotic trajectory of the mobile robot

5 Discussion

The figures 2, 3, 4 and 5 shows the effectiveness of the closed-loop flatness control which allows the mobile robot to follow the desired reference trajectory properly. By ensuring a good tracking of the trajectory the mobile robot can move by avoiding the static obstacles with minimum of energy and by choosing the optimal trajectory .

Figure 7 shows the behavior of the Lorenz chaotic system in the cartesian plan.

In figures 9, 10 and 12 we present the good effeteness of chaotic trajectory tracking , and we ensure that the robot reaches its desired trajectory as well. Even more, as depicted in figure 11 we can show the robustness of the control strategy with chaotic phenomena and in presence of obstacles.

Next, some endeavours for uncovering the chaotic behaviour of robots were presented. Chaos can be employed for analyzing robotics arms and chaos quantifiers can be used for analyzing chaotic dynamics in robot-environment interaction.

6 Conclusion

This article has described the path planning and the flatness based tracking control of a wheeled mobile robot. The flatness based approach to trajectory control and optimal trajectory tracking offers a fast alternative to classical control for such robots. Having determined the flat output of the mobile robot, trajectory control was determined with reasonable accuracy. Secondly, we have presented a chaotic phenomenon tuned to the behavior of the autonomous mobile robot, so we solved the problem related to this phenomenon using the differential flatness method.

In recent years, the discovery of chaos has attracted much interest among investigators. Deterministic chaos leads to a quantitative analysis, which is the essence of science. Despite several efforts to find evidence of chaotic dynamics in robotics, useful applications of deterministic chaos in robotics have rarely been studied.

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