Robust Tracking Controller for Quadrotor Based on Flatness and High Gain Observer

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***Abstract*—This paper proposes a robust tracking controller based on the state estimation for a quadrotor. Using the differential flatness theory, it is demonstrated that the quadrotor model can be changed to a linearized form which facilitate the creation of a state feedback controller. Since some state vectors of the obtained linearized model cannot be measured directly, a high gain observer is implemented to estimate them. After that, utilizing the estimation state obtained by the latter observer, a new guidance law is developed for the quadrotor enabling a robust tracking to the desired trajectory despite the existence of unmesurable states. The numerical simulation of the quadrotor system is done in order to evaluate the performance of the robust tracking control scheme.**

***Index Terms*—Differential flatness, tracking controller, high-gain observers, quadrotor.**

I. Introduction

Recently, the use and development of quadrotors become very important compared with other flying vehicles. This superiority is due to the simplicity of their mechanical structure, good maneuverability and low speed flight. The quadrotor has been applied in many areas such as transportation, military interdiction, rescue, surveillance etc. Despite these advantages, the quadrotor has a strongly nonlinear model with coupling multi-variables. Therefore, the trajectory planning and the tracking problem for these latter become more complex.

Currently, the differential flatness property introduced by Fliess [1] has proven to be a good tool to ameliorate the trajectory planning and to create tracking controllers for linear and nonlinear systems. Thus, with flatness, we can express all the trajectories of the system as a function of the flat outputs and their derivatives. Consequently, we can develop a flatness control which allows the system to pass from an initial state to an end state. In the last decade, the flatness property has been extensively used for the planning and tracking of quadrotor trajectory. In [2], Jing Yu proposed an optimal trajectory generation algorithm based on transcription method, flatness and B-spline curve. Consequently, based on the flatness property, the number of optimization variables decreased, which facilitated the resolution task for the optimization solver. In [3], Lu developed a trajectory planner based on the Bezier polynomials to resolve the online optimization problem and on backstepping control to resolve the trajectory tracking problem. In [4], José combined a flatness and predictive control strategy to ensure an online trajectory tracking.

Although those tracking control schemes can ameliorate the trajectory tracking result of the quadrotor, all of them are developed on assuming that all of the states are measurable which is not always true in practice flight. In addition, the augmentation number of sensors complicates the implementation of the system in real application, hence the need for a good observer to estimate the quadrotor state.

In the last decade the high gain observer [5] appear as a good solution to estimate the system state under measurement noise. Therefore, this observer has the same structure of the Luenberger observer. The advantages of the high gain observer have been utilized to develop different robust tracking controllers, which can be applied strongly in a lot of applications such as, the sliding mode control for nonlinear system [6], the adaptive control of flexible-joints surgical robot [7], the tracking feedback controller for wheeled mobile robot [8], the robust integral backstepping controller for permanent magnet synchronous motor [9], and the feedback Linearization of a robot manipulator system [10].

In this paper, the contribution consists in creating a robust tracking law for the quadrotor based on a flatness and a high gain observer. Utilizing the flatness property, it is proven that the quadrotor model can be changed to the Burnovsky canonical system [11]. According to this obtained linearized model, a state feedback controller is developing permitting an exact trajectory tracking. Other problems are treated in the creation of this feedback control consist on the estimation of the non-measurable state variables in the quadrotor model. To deal with such a problem, a high gain observer is proposed. After that, based the observer result, a robust guidance law is implemented for the quadrotor permitting an accurate tracking to desired trajectory despite the existence of un-measurable states and measurement noise.

This article is organized as follows. In section II, we present the quadrotor model. In section III, we define the tracking controller for the quadrotor. In section IV, we present a robust tacking controller for the quadrotor. Finally, section V deals with the simulation results.

II. QUADROTOR MODEL

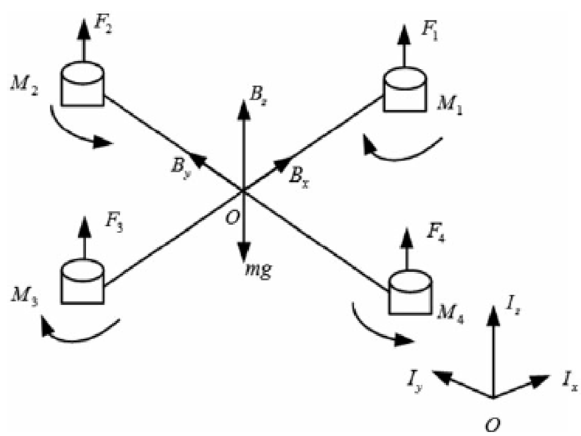


Figure.1 Quadrotor aircraft scheme.

A quadrotor (Figure 1) is an aircraft with four engines installed on a cross usually made of carbon fiber. In [12], the principle of quadrotor flight was described. In the literature, a lot of work has been done to determine the dynamic model of the quadrotor. In this article, we consider the commonly employed quadrotor model obtained via the Newton-Euler equations as follows [13]:

$$\ddot{x} = \frac{\tau_1}{m}(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) \tag{1}$$

$$\ddot{y} = \frac{\tau_1}{m}(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) \tag{2}$$

$$\ddot{z} = \frac{\tau_1}{m}(\cos\theta\cos\phi) - g \tag{3}$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{l}{I_y}\tau_2 \tag{4}$$

$$\ddot{\phi} = \dot{\theta}\dot{\psi}(\frac{I_z - I_y}{I_x}) + \frac{l}{I_x}\tau_3 \tag{5}$$

$$\ddot{\psi} = \dot{\phi}\dot{\theta}(\frac{I_y - I_x}{I_z}) + \frac{1}{I_z}\tau_4 \tag{6}$$

where $X_p = [x, y, z]$ are the coordinates of the quadrotor center, $\Theta = [\theta, \phi, \psi]$ are the Euler angles, m represents the mass, g is the acceleration and l is the distance from the center of gravity to each rotor. The moments of inertia along the directions x, y and z are defined by Ix, Iy and Iz. Moreover, τ_1, τ_2, τ_3 and τ_4 are the controlled input. The quadrotor model stands as a relatively complex model to deal with. Thereby, we suppose that when the quadrotor flies towards the target $\psi = 0$. In this condition, the quadrotor model (1-6) can be defined as follows:

$$\ddot{x} = \frac{\tau_1}{m} sin\theta\cos\phi \tag{7}$$

$$\ddot{y} = \frac{-\tau_1}{m} sin\phi \tag{8}$$

$$\ddot{z} = \frac{\tau_1}{m}\cos\theta\cos\phi - g \tag{9}$$

$$\ddot{\theta} = \frac{l}{I_y}\tau_2 \tag{10}$$

$$\ddot{\phi} = \frac{l}{I_x}\tau_3 \tag{11}$$

III. TRACKING CONTROL

In this section, a flatness-based tracking control is created for the quadrotor. This control law permits the quadrotor system to follow the desired reference trajectories. Consider The nonlinear system

$$\dot{x} = f(x, u), x \in \mathbb{R}^n, u \in \mathbb{R}^m \tag{12}$$

The system defined by equation (12) is differentially flat if it is possible to have the following outputs:

$$\sigma = \xi_1(x, u, \dot{u}, ..., u^{(r-1)}) \tag{13}$$

and

$$x = \xi_2(\sigma, \dot{\sigma}, \ddot{\sigma}...., \sigma^{(\beta)}) \tag{14}$$

$$u = \xi_3(\sigma, \dot{\sigma}, \ddot{\sigma}..., \sigma^{(\beta+1)}) \tag{15}$$

where β and r are a real number and ξ_1, ξ_2 and ξ_3 are a functions.

The quadrotor model is a differentially flat system whose flat outputs are given by $\sigma = [\sigma_{11}, \sigma_{21}, \sigma_{31}]^T = [x, y, z]^T$. So, all the state and control inputs for the quadrotor can be expressed as a function of the flat outputs σ and their derivatives as follows:

$$\theta = \arctan(\frac{\ddot{\sigma}_1}{g + \ddot{\sigma}_3}) \tag{16}$$

$$\phi = \arcsin(\frac{-\ddot{\sigma}_2}{\sqrt{\ddot{\sigma}_1^2 + \ddot{\sigma}_2^2 + (g + \ddot{\sigma}_3)^2}}) \tag{17}$$

$$\tau_1 = m\sqrt{\ddot{\sigma}_1^2 + \ddot{\sigma}_2^2 + (g + \ddot{\sigma}_3)^2} \tag{18}$$

$$\tau_2 = \frac{l}{I_y}(\frac{\ddddot{\sigma}_1}{g + \ddot{\sigma}_3} - \frac{\ddot{\sigma}_1\ddddot{\sigma}_3}{(g + \ddot{\sigma}_3)^2} - 2\frac{\dddot{\sigma}_1\dddot{\sigma}_3}{(g + \ddot{\sigma}_3)^2} + 2\frac{\ddot{\sigma}_1(\dddot{\sigma}_3)^2}{(g + \ddot{\sigma}_3)^3}) \tag{19}$$

$$\tau_3 = \frac{l}{I_x}\left(\frac{(\ddot{u}_1\sigma_2 - u_1\dddot{\sigma}_2)(u_1\sqrt{u_1^2-\ddot{\sigma}_2^2}) - (\dot{u}_1\ddot{\sigma}_2 - u_1\dddot{\sigma}_2)(\dot{u}_1\sqrt{(u_1^2-\ddot{\sigma}_2^2)})}{u_1^2(u_1^2-\ddot{\sigma}_2^2)}\right) + \frac{l}{I_x}\left(\frac{(u_1(u_1^2(u_1^2-\ddot{\sigma}_2^2)^{1/2}(u_1\dot{u}_1 - \ddot{\sigma}_2\dddot{\sigma}_2)))}{u_1^2(u_1^2-\ddot{\sigma}_2^2)}\right) \quad (20)$$

where

$$u_1 = \sqrt{\ddot{\sigma}_1^2 + \ddot{\sigma}_2^2 + (g + \ddot{\sigma}_3)^2} \quad (21)$$

The flatness property allows computing an endogenous feedback linearization that transforms the non linear system in a controllable linear system. Because the relationship between the control input vector, (τ_1, τ_2, τ_3), and the flat output's highest derivatives is not invertible, it is necessary to create a second order dynamic prolongation of τ_1. Considering τ_1 and $\dot{\tau}_1$ as an additional state, the new state and control of the prolonged quadrotor systems are given by $X = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \theta, p, \phi, q, \tau_1, \dot{\tau}_1]^T$ and $U = [\ddot{\tau}_1, \tau_2, \tau_3]^T$. To obtain the invertible relation between the prolonged inputs and the higher derivatives of the flat outputs, we differentiate the equations (7), (8), (9) until the input terms $\ddot{\tau}_1$, τ_2, τ_3 appear as follows:

$$\begin{bmatrix} \ddddot{\sigma}_{11} \\ \ddddot{\sigma}_{21} \\ \ddddot{\sigma}_{31} \end{bmatrix} = A + B \begin{bmatrix} \ddot{\tau}_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \quad (22)$$

with

$$A = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, B = \begin{bmatrix} \mu_{x1} & \mu_{x2} & \mu_{x3} \\ \mu_{y1} & \mu_{y2} & \mu_{y3} \\ \mu_{z1} & \mu_{z2} & \mu_{z3} \end{bmatrix}$$

where a_x, a_y, a_z, μ_{x1}, μ_{x2}, μ_{x3}, μ_{x4}, μ_{y1}, μ_{y2}, μ_{y3}, μ_{z1}, μ_{z2} and μ_{z3} are written as a function of the flat outputs σ and their derivatives. According to equation (22), the feedback law that linearizes the quadrotor system can be defined as follows:

$$\begin{bmatrix} \ddot{\tau}_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = B^{-1} \begin{bmatrix} \ddddot{\sigma}_{11} \\ \ddddot{\sigma}_{21} \\ \ddddot{\sigma}_{31} \end{bmatrix} - A \quad (23)$$

When substituting the control input defined by equations (23) in equation (22), we obtain the state-space Brunovsky form (BF) as follows:

$$BF_1 \begin{cases} \dot{\sigma}_{11} = \sigma_{12} \\ \dot{\sigma}_{12} = \sigma_{13} \\ \dot{\sigma}_{13} = \sigma_{14} \\ \dot{\sigma}_{14} = v_x \end{cases} \quad (24)$$

$$BF_2 \begin{cases} \dot{\sigma}_{21} = \sigma_{22} \\ \dot{\sigma}_{22} = \sigma_{23} \\ \dot{\sigma}_{23} = \sigma_{24} \\ \dot{\sigma}_{24} = v_y \end{cases} \quad (25)$$

$$BF_3 \begin{cases} \dot{\sigma}_{31} = \sigma_{32} \\ \dot{\sigma}_{32} = \sigma_{33} \\ \dot{\sigma}_{33} = \sigma_{34} \\ \dot{\sigma}_{34} = v_z \end{cases} \quad (26)$$

where v_x, v_y and v_z are an appropriate feedback controller that permits the flat output σ_{11}, σ_{21} and σ_{31} to track the desirable reference trajectories σ_{xd}, σ_{yd}, σ_{zd}, respectively. The feedback controller is defined as follows:

$$\begin{aligned} v_x = \ddddot{\sigma}_{xd} &+ K_{x4}(\dddot{\sigma}_{xd} - \sigma_{14}) + K_{x3}(\ddot{\sigma}_{xd} - \sigma_{13}) \\ &+ K_{x2}(\dot{\sigma}_{xd} - \sigma_{12}) + K_{x1}(\sigma_{xd} - \sigma_{11}) \end{aligned} \quad (27)$$

$$\begin{aligned} v_y = \ddddot{F}_{yd} &+ K_{y4}(\dddot{F}_{yd} - F_{24}) + K_{y3}(\ddot{F}_{yd} - F_{23}) \\ &+ K_{y2}(\dot{F}_{yd} - F_{22}) + K_{y1}(F_{yd} - F_{21}) \end{aligned} \quad (28)$$

$$\begin{aligned} v_z = \ddddot{F}_{zd} &+ K_{z4}(\dddot{F}_{zd} - F_{34}) + K_{z3}(\ddot{F}_{zd} - F_{33}) \\ &+ K_{z2}(\dot{F}_{zd} - F_{32}) + K_{z1}(F_{zd} - F_{31}) \end{aligned} \quad (29)$$

The feedback gains K_{x1}, K_{x2}, K_{x3}, K_{x4}, K_{y1}, K_{y2}, K_{y3}, K_{y4}, K_{z1}, K_{z2}, K_{z3} and K_{z4} can be chosen so that the characteristic polynomial associated to each flat output tracking error is Hurwitz. The flatness control defined by equation (23) cannot allow an asymptotic robust tracking of the trajectory under the existence of un-measurable state and measurement noise, hence the need for a robust tracking controller that will be designed based on an observer that estimates all the state despite the existence of measurement noise.

IV. Robust Tracking Control

The development of the control laws (22) necessitates the knowledge (measurement) of the state σ and its derivatives. Therefore, a high gain observer is designed. The Burnovsky systems defined by equations (24-26) can be written in a matrix form as follows:

$$\dot{\sigma}_1 = A_1\sigma_1 + B_1 v_x \quad (30)$$

$$\dot{\sigma}_2 = A_2\sigma_2 + B_2 v_y \quad (31)$$

$$\dot{\sigma}_3 = A_3\sigma_3 + B_3 v_z \quad (32)$$

where
$\sigma_1 = [\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}]^T$, $\sigma_2 = [\sigma_{21}, \sigma_{22}, \sigma_{23}, \sigma_{24}]^T$,
$\sigma_3 = [\sigma_{31}, \sigma_{32}, \sigma_{33}, \sigma_{34}]^T$,

$$A_1 = A_2 = A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_1 = B_2 = B_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The high-gain observer proposed to estimate the state of the Burnovsky form (24-26) are defined as follows:

$$\dot{\hat{\sigma}}_1 = A_1\hat{\sigma}_1 + B_1 v_x + L_1 C_1(\sigma_1 - \hat{\sigma}_1) \quad (33)$$

$$\dot{\hat{\sigma}}_2 = A_2\hat{\sigma}_2 + B_2 v_y + L_2 C_2(\sigma_2 - \hat{\sigma}_2) \quad (34)$$

$$\dot{\hat{\sigma}}_3 = A_3\hat{\sigma}_3 + B_3 v_z + L_3 C_3(\sigma_3 - \hat{\sigma}_3) \quad (35)$$

where $C_1 = C_2 = C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$. The gain matrix L_i has the following form:

$$L_1 = \begin{bmatrix} \frac{\Gamma_{11}}{\epsilon} & \frac{\Gamma_{12}}{\epsilon^2} & \frac{\Gamma_{13}}{\epsilon^3} & \frac{\Gamma_{14}}{\epsilon^4} \end{bmatrix}^T \quad (36)$$

$$L_2 = \begin{bmatrix} \frac{\Gamma_{21}}{\epsilon} & \frac{\Gamma_{22}}{\epsilon^2} & \frac{\Gamma_{23}}{\epsilon^3} & \frac{\Gamma_{24}}{\epsilon^4} \end{bmatrix}^T \quad (37)$$

$$L_3 = \begin{bmatrix} \frac{\Gamma_{31}}{\epsilon} & \frac{\Gamma_{32}}{\epsilon^2} & \frac{\Gamma_{33}}{\epsilon^3} & \frac{\Gamma_{34}}{\epsilon^4} \end{bmatrix}^T \quad (38)$$

where ϵ is a small positive number. In addition, the parameters $\Gamma_{11}, \Gamma_{12}, \Gamma_{13}, \Gamma_{14}, \Gamma_{21}, \Gamma_{22}, \Gamma_{23}, \Gamma_{24}, \Gamma_{31}, \Gamma_{32}, \Gamma_{33}$ and Γ_{34} are selected in order that the roots of the following equation:

$$s^4 + \Gamma_{11}s^3 + \Gamma_{12}s^2 + \Gamma_{13}s + \Gamma_{14} = 0 \quad (39)$$

$$s^4 + \Gamma_{21}s^3 + \Gamma_{22}s^2 + \Gamma_{23}s + \Gamma_{24} = 0 \quad (40)$$

$$s^4 + \Gamma_{31}s^3 + \Gamma_{32}s^2 + \Gamma_{33}s + \Gamma_{34} = 0 \quad (41)$$

have a real negative part.
Based on the high observer result defined by equation (33-35), a new estimated feedback controller can be obtained when replacing the states by their estimation in the feedback controller defined by equations (27-29) as follows:

$$\begin{aligned}\hat{v}_x = \dddot{\sigma}_{xd} + K_{x4}(\ddot{\sigma}_{xd} - \hat{\sigma}_{14}) + K_{x3}(\ddot{\sigma}_{xd} - \hat{\sigma}_{13}) \\ +K_{x2}(\dot{\sigma}_{xd} - \hat{\sigma}_{12}) + K_{x1}(\sigma_{xd} - \hat{\sigma}_{11})\end{aligned} \quad (42)$$

$$\begin{aligned}\hat{v}_y = \dddot{\sigma}_{yd} + K_{y4}(\ddot{\sigma}_{yd} - \hat{\sigma}_{24}) + K_{y3}(\ddot{\sigma}_{yd} - \hat{\sigma}_{23}) \\ +K_{y2}(\dot{\sigma}_{yd} - \hat{\sigma}_{22}) + K_{y1}(\sigma_{yd} - \hat{\sigma}_{21})\end{aligned} \quad (43)$$

$$\begin{aligned}\hat{v}_z = \dddot{\sigma}_{zd} + K_{z4}(\ddot{\sigma}_{zd} - \hat{\sigma}_{34}) + K_{z3}(\ddot{\sigma}_{zd} - \hat{\sigma}_{33}) \\ +K_{z2}(\dot{\sigma}_{zd} - \hat{\sigma}_{32}) + K_{z1}(\sigma_{zd} - \hat{\sigma}_{31})\end{aligned} \quad (44)$$

When integrating the estimated feedback defined by equation (42-44) in the flatness-based tracking control (23), we obtain the robust tracking control applied to the quadrotor as follows:

$$\begin{bmatrix}\ddot{\hat{\tau}}_1 \\ \hat{\tau}_2 \\ \hat{\tau}_3\end{bmatrix} = \hat{B}^{-1}\begin{bmatrix}\hat{v}_x \\ \hat{v}_y \\ \hat{v}_z\end{bmatrix} - \hat{A} \quad (45)$$

$$\hat{A} = \begin{bmatrix}\hat{a}_x \\ \hat{a}_y \\ \hat{a}_z\end{bmatrix}, B = \begin{bmatrix}\hat{\mu}_{x1} & \hat{\mu}_{x2} & \hat{\mu}_{x3} \\ \hat{\mu}_{y1} & \hat{\mu}_{y2} & \hat{\mu}_{y3} \\ \hat{\mu}_{z1} & \hat{\mu}_{z2} & \hat{\mu}_{z3}\end{bmatrix}$$

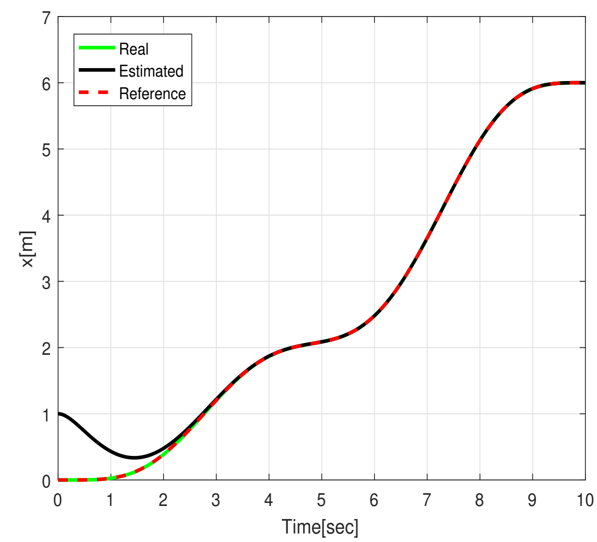
where $\hat{a}_x, \hat{a}_y, \hat{a}_z, \hat{\mu}_{x1}, \hat{\mu}_{x2}, \hat{\mu}_{x3}, \hat{\mu}_{x4}, \hat{\mu}_{y1}, \hat{\mu}_{y2}, \hat{\mu}_{y3}, \hat{\mu}_{z1}, \hat{\mu}_{z2}$ and $\hat{\mu}_{z3}$ are a function depending on the estimated flat outputs $\hat{\sigma}$ and their derivatives.

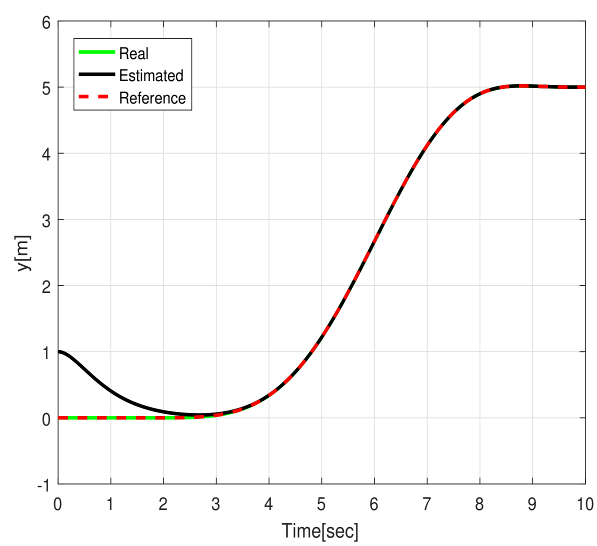
V. SIMULATION AND RESULTS

In this section, the proposed robust tacking control based on flatness and high gain observer is implemented to demonstrate its effectiveness. Then we consider the AR-Drone quadrotor whose parameters are given by :
$I_x = I_y = 0.003\ Kg.m^2$, $I_z = 0.006\ Kg.m^2$, $m = 0.4\ Kg$ $l = 0.3\ m$, $g = 9.81\ m.s^{-2}$.
Before dealing with the control and the observer, a reference trajectory is generated for the quadrotor permits its movement from the initial state $X(0) = [0,0,0,0,0,0,0,0,0,0]^T$ to the final state $X(10) = [6,0,5,0,0,0,0,0,0,0]^T$. Based on the flatness property of quadrotor system, the initial conditions for the desired trajectories are $\sigma_{xd}(0) = \dot{\sigma}_{xd}(0) = \ddot{\sigma}_{xd}(0) = \sigma_{yd}(0) = \dot{\sigma}_{yd}(0) = \ddot{\sigma}_{yd}(0) = \sigma_{zd}(0) = \dot{\sigma}_{zd}(0) = \ddot{\sigma}_{zd}(0) = 0$ and the final conditions are $\sigma_{xd}(10) = \dot{\sigma}_{xd}(10) = \dot{\sigma}_{yd}(10) = \ddot{\sigma}_{yd}(10) = \dot{\sigma}_{zd}(10) = \ddot{\sigma}_{zd}(10) = 0, \sigma_{xd}(10) = 6, \sigma_{yd}(10) = 5$.
Thereby, any curve that satisfies this condition can be used as a desired trajectory for the quadrotor. In our case, we utilize the B-spline [14] curve of order 8 as a suitable function to approximate the flat output as follows:

$\sigma_{id}(t) = \delta_{i0}(1-t)^8 + 8\delta_{i1}(1-t)^7t + 28\delta_{i2}(1-t)^6t^2P + 56\delta_{i3}(1-t)^5t^3 + 70\delta_{i4}(1-t)^4t^4 + 56\delta_{i5}(1-t)^3t^5 + 28\delta_{i6}(1-t)^2t^6 + 8\delta_{i7}(1-t)t^7 + \delta_{i8}t^8$.

where δ_{ij} $i = 1,2,3, j = 0,1,2,3,4,5,6,7,8$ are the variable parameters of the B-spline curve [14]. To show more the robustness of the proposed tracking controller, it is considered that the quadrotor undergoes a parametric variation of 30% respectively in m and l. In addition, we assume that the quadrotor system is subjected to Gaussian noise. The high gain observer parameters are selected as: $\epsilon = 0.005$, $\Gamma_{14} = \Gamma_{24} = \Gamma_{24} = 81$, $\Gamma_{13} = \Gamma_{23} = \Gamma_{23} = 36$, $\Gamma_{12} = \Gamma_{22} = \Gamma_{32} = 27$ and $\Gamma_{11} = \Gamma_{21} = \Gamma_{31} = 12$. The controller parameters are selected as follows: $K_{x1} = K_{y1} = K_{z1} = 625$, $K_{x2} = K_{y2} = K_{z2} = 500$, $K_{x3} = K_{y3} = K_{z2} = 150$, $K_{x4} = K_{y4} = K_{z2} = 20$. Figure 2, 3, 4 and 5 depict the output response of the system.





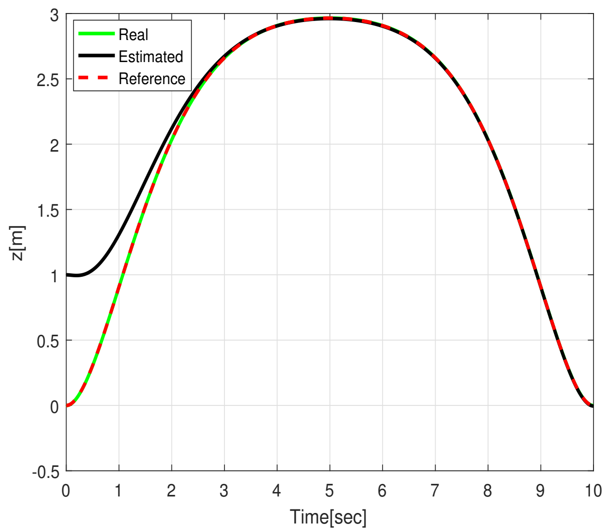


Figure.2 Simulation results for quadrotor position

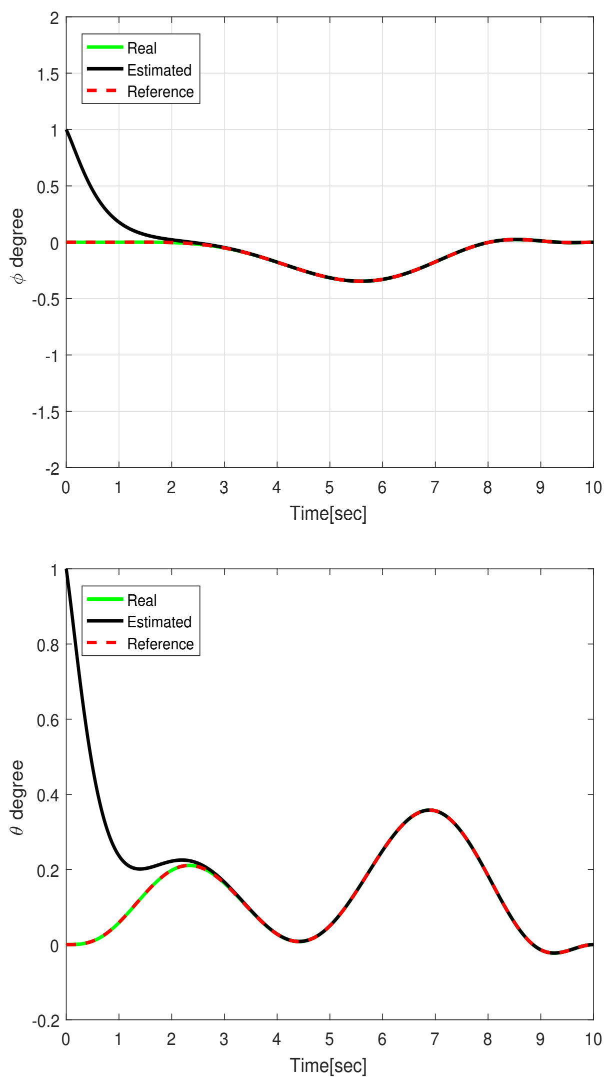


Figure.3 Simulation results for quadrotor attitude

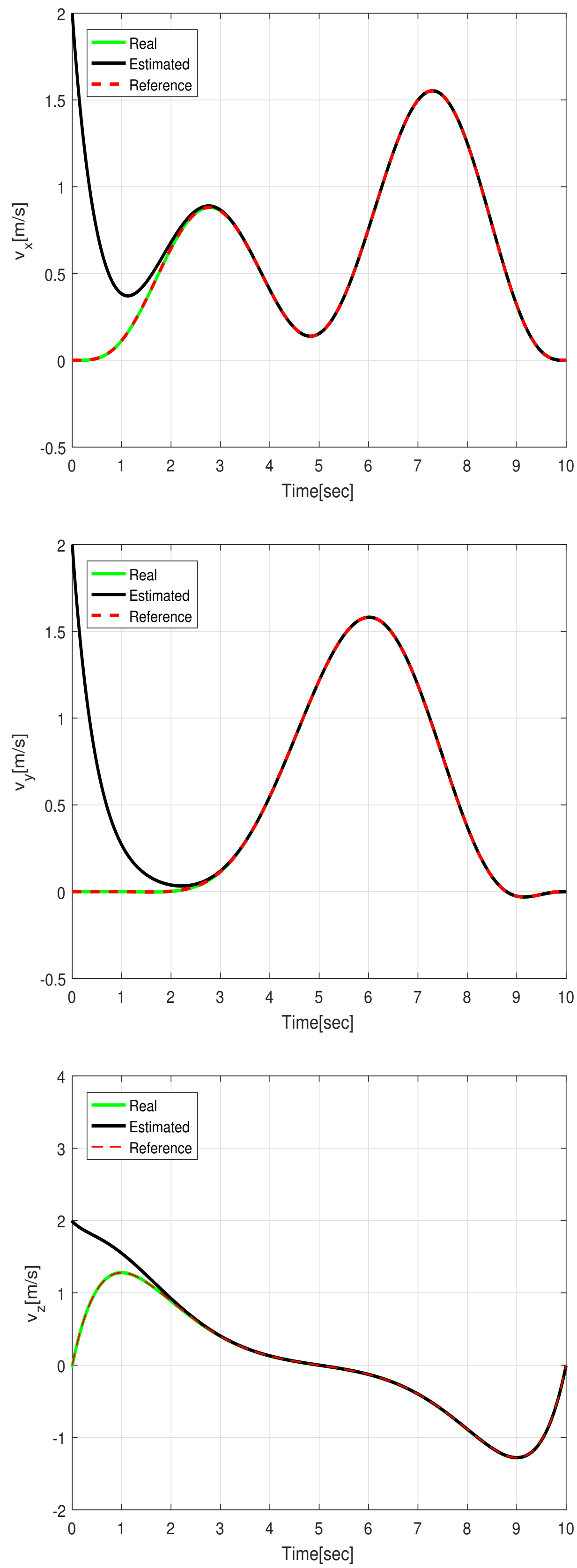
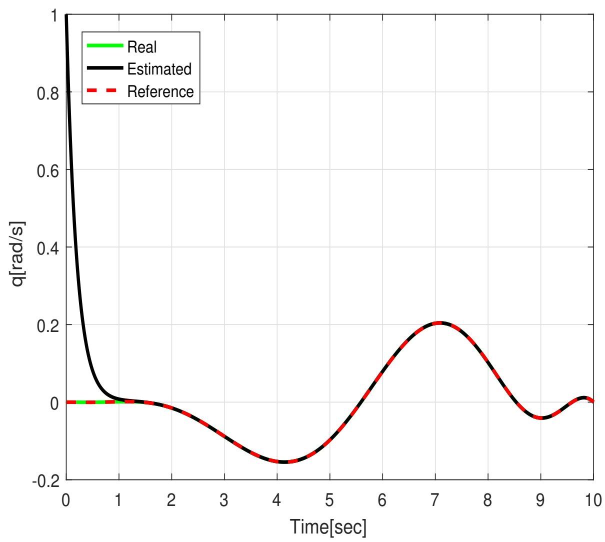


Figure.4 Simulation results for quadrotor velocity.



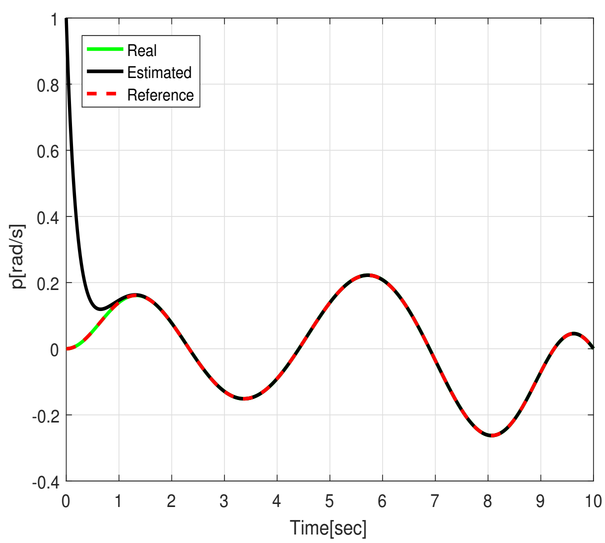


Figure.5 Simulation results for quadrotor angular velocity.

From Figures 2-5, it can be seen firstly that quadrotor state can be estimated by the high gain observer under the existence of measurement noise. In addition, Figures 2-5 show that the quadrotor can successfully follow the desired reference. Consequently, we can deduce that the proposed guidance law based on flatness and a high gain observer improves the tracking performance for the quadrotor despite the existence of un-measurable states and measurement noise.

VI. Conclusion

The tracking trajectory problem for the quadrotor is studied in this article. Based on the flatness property that the system presents and on the high gain observer, a new guidance law is proposed for the quadrotor in order to improve its tracking performance. The simulation results indicate that the proposed tracking control scheme is robust against parameter uncertainties and measurement noise. In Future work, we will consider the existence of disturbance in the quadrotor model.

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