Sliding Mode Control of Quadrotor Based on Differential Flatness

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***Abstract*— In this work, the differential flatness property and the sliding mode controller design are proposed for a quadrotor in order to track a predefined reference trajectory despite the presence of uncertain parameters. Firstly, differential flatness is utilized to resolve the problem for trajectory generation and tracking for the quadrotor. Next, a sliding controller is combined with a flatness design to guarantee the robustness of the tracking strategy. The numerical simulations of quadrotor system are done in order to evaluate the performance of the suggested control scheme.**

I. INTRODUCTION

In recent years, special attention has been paid to drones because they can successively follow trajectory and stationary flight. This gives them many practicable applications such as military interdiction, transportation and surveillance. In this sense, intensive research efforts have been devoted to the quadrotor helicopters because of their advantages over the conventional drones. This dominance is due to the simplicity of the mechanical structure, the good maneuverability and low speed flight. Despite these advantages, the quadrotor has a strongly nonlinear model with coupling multi-variables. Therefore, the trajectory planning and the tracking problem for quadrotor become more complex.

Recently, the flatness property introduced by Fliess [1] has proven to be a good tool to enhance the trajectory planning and to design tracking controllers for linear and nonlinear systems. Thus , the flatness can express all the trajectories of the system as a function of the flat outputs and their derivatives. Consequently, we can develop an open-loop control law which allow the system to pass from an initial state to an end state.

In the last decade, flatness property has been extensively used for the planning and tracking of quadrotor trajectory. In [2], Jing Yu proposed an optimal trajectory generation approach based on the transcription method, the flatness and the b-spline curve. As a result, the flatness allows the decrease in the variable number of optimization trajectory problem to have a more computational performance. Lu [3], developed a trajectory planner based on the Bezier polynomials to resolve online optimization problem and a backstepping control to resolve the trajectory tracking problem. José [4] combined a flatness and predictive control strategy to ensure online trajectory tracking.

Although those control strategies can improve the trajectory tracking performance, all of them are designed based on exact model and do not compute parameter uncertainties in quadrotor model. Hence, the sliding control is combined with flatness to improve the robustness of the tracking strategy. The sliding mode control developed by Utkin [5] was a special type of a variable structure control. Its principal idea consisted, firstly in converge the system states towards a sliding surface called a sliding surface which depended on a set of static relationships between state variables. It consisted secondly in designing a discontinuous control law that permits the stabilization of the system states on such a surface. The advantage of this approach its insensitivity to parametric uncertainties and the model errors. Despite this advantage, the discontinuity of the sliding control causes chattering [6], which can excite the high frequencies of the process and damage it.

On the other hand, there exists a lot of work utilising the sliding mode to control the quadrotors. In [7], Benallegue suggested a feedback linearization controller based on a high-order sliding mode observer. The observer role consisted in the estimation of the external disturbance effect. Runcharoon [8] developed a sliding control for the quadrotor. The controller is composed of two parts, the sliding mode was utilized to control the attitude of the system and the proportional-derivative controller was done so as to stabilize the horizontal position of the system. In [9], Sudhir combined adaptive and sliding controllers in order to obtain a robust tracking strategy in spite the existence of disturbances with unknown bounds. In [10], Mallavalli developed an integral terminal sliding mode control to ensure the trajectory tracking to the quadrotor subject to actuator faults. In [11], Yeh put forward a robust attitude controller based on the sliding mode controller and the fuzzy inference mechanism for the quadrotor in the existence of white noise interference.

In this paper, the contribution consists in creating a robust guidance law for a quadrotor based on flatness and a sliding controller. So, the flatness ensures the tracking and the reference planning and the sliding controller improves the robustness of the tracking guidance law.

This article is organized as follows. In section II, we present the quadrotor model. In section III, we define the flatness control for the quadrotor. In section IV, we present the sliding approach based on flatness. Finally, section V deals with the simulation results.

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II. QUADROTOR MODEL

The quadrotor in Figure.1 is an aircraft with four engines installed on a cross usually made of carbon fiber. The front and rear engines rotate clockwise while the right and left engines rotate in the opposite direction. A lot of work has been done on the mathematical modeling of a quadrotor and the equations of motion are well established. In this article, we consider the commonly employed quadrotor model obtained via the Lagrange approach as follow [12]:

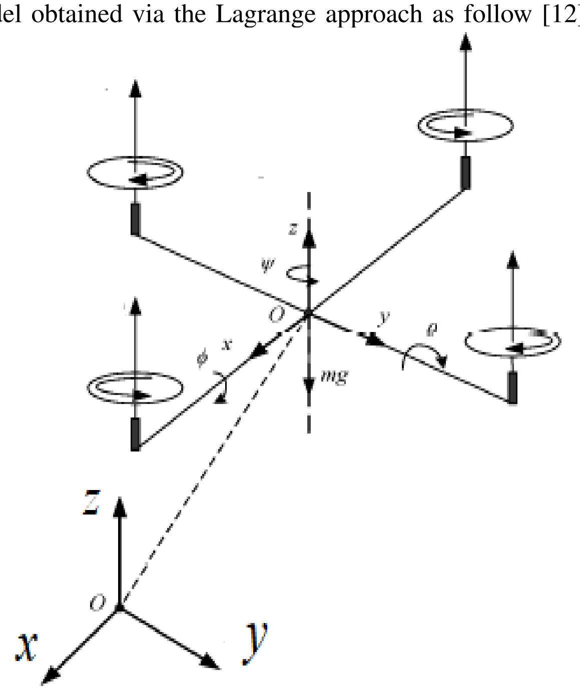


Figure.1 Quadrotor aircraft scheme.

$$\ddot{x}(t) = \frac{u_1}{M}(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) \quad (1)$$

$$\ddot{y}(t) = \frac{u_1}{M}(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) \quad (2)$$

$$\ddot{z}(t) = \frac{u_1}{M}(\cos\theta\cos\phi) - g \quad (3)$$

$$\ddot{\theta}(t) = \dot{\phi}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{l}{I_y}u_2 \quad (4)$$

$$\ddot{\phi}(t) = \dot{\theta}\dot{\psi}(\frac{I_z - I_y}{I_x}) + \frac{l}{I_x}u_3 \quad (5)$$

$$\ddot{\psi}(t) = \dot{\phi}\dot{\theta}(\frac{I_y - I_x}{I_z}) + \frac{l}{I_z}u_4 \quad (6)$$

where x,y and z are the coordinates of the quadrotor center. M represents the mass, θ, ϕ and ψ are the Euler angles. g is the acceleration, l is the distance from the center of gravity to each rotor. The moments of inertia along the directions x, y and z are defined by Ix, Iy and Iz. Moreover, u_1, u_2, u_3 and u_4 are the controlled input. Quadrotor model stands as a relatively complex model to deal with. Then, we assume that θ and ϕ are very small and when the quadrotor flies to-wards the target $\psi = 0$. In this condition, the quadrotor model (1-6) can be rewritten as follows:

$$\ddot{x}(t) = \frac{\theta u_1}{M} \quad (7)$$

$$\ddot{y}(t) = \frac{-\phi u_1}{M} \quad (8)$$

$$\ddot{z}(t) = \frac{u_1}{M} - g \quad (9)$$

$$\ddot{\theta}(t) = \frac{l}{I_y}u_2 \quad (10)$$

$$\ddot{\phi}(t) = \frac{l}{I_x}u_3 \quad (11)$$

where the state system of the quadrotor is $X = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \theta, \dot{\theta}, \phi, \dot{\phi}]^T$ and the control input is $U = [u_1, u_2, u_3]^T$.

III. FLATNESS CONTROL

The following nonlinear system

$$\dot{x} = f(x, u), x \in \mathbb{R}^n, u \in \mathbb{R}^m \quad (12)$$

is differentially flat if we find the following outputs:

$$F = \xi(x, u, \dot{u}, ..., u^{(r-1)}) \quad (13)$$

and

$$x = \gamma(F, \dot{F}, \ddot{F}...., F^{(\alpha)}) \quad (14)$$

$$u = \gamma(F, \dot{F}, \ddot{F}..., F^{(\alpha)}) \quad (15)$$

where α and r are finite multi-indices and ξ and γ are smooth vector functions of the output vector F .
The flatness property allows computing an endogenous feedback linearization and a diffeomorphism that transforms the closed-loop system in a controllable linear system whose flat outputs constitute the state vector .
It can be shown that the quadrotor model is a differentially flat system whose flat outputs are given by $F_1 = z$, $F_2 = x$, $F_3 = y$. The control of the vertical position can be obtained by using equation (9) as follows:

$$u_1 = M(g + \ddot{F}_1) \quad (16)$$

By introducing (16), the quadrotor system (7)-(11) is written as follows:

$$\ddot{x}(t) = \theta(g + \ddot{F}_1) \quad (17)$$

$$\ddot{y}(t) = -\phi(g + \ddot{F}_1) \quad (18)$$

$$\ddot{z}(t) = \ddot{F}_1 \quad (19)$$

$$\ddot{\theta}(t) = \frac{l}{I_y}u_2 \quad (20)$$

$$\ddot{\phi}(t) = \frac{l}{I_x}u_3 \quad (21)$$

To obtain u_2 and u_3, we differentiate (17) and (18) until the input terms $u_2 = \frac{Iy}{l}\ddot{\theta}$ and $u_3 = \frac{Ix}{l}\ddot{\phi}$ appear as follows :

$$\ddddot{x}(t) = \ddddot{z}\theta + 2\dot{\theta}\dddot{z} + \ddot{\theta}(g + \ddot{z}) \quad (22)$$

$$\ddddot{y}(t) = -\phi\ddddot{z} - 2\dot{\phi}\dddot{z} - \ddot{\phi}(g + \ddot{z}) \quad (23)$$

Note that the parameterization of θ , $\dot{\theta}$ and ϕ ,$\dot{\phi}$ in function of the flat outputs is as follows:

$$\theta = \frac{\ddot{F}_2}{g+\ddot{F}_1} \tag{24}$$

$$\phi = \frac{-\ddot{F}_3}{g+\ddot{F}_1} \tag{25}$$

$$\dot{\theta} = \frac{\dddot{F}_2(g+\ddot{F}_1) - \dddot{F}_1\ddot{F}_2}{(g+\ddot{F}_1)^{(2)}} \tag{26}$$

$$\dot{\phi} = \frac{-\dddot{F}_3(g+\ddot{F}_1) + \dddot{F}_1\ddot{F}_3}{(g+\ddot{F}_1)^{(2)}} \tag{27}$$

The input terms u_2 and u_3 can be defined as follows:

$$u_2 = \frac{Iy}{l}\left(\frac{\ddddot{F}_2}{g+\ddot{F}_1} - \frac{\ddot{F}_2\ddddot{F}_1}{(g+\ddot{F}_1)^2} - 2\frac{\dddot{F}_2\dddot{F}_1}{(g+\ddot{F}_1)^2} + 2\frac{\ddot{F}_2(\dddot{F}_1)^2}{(g+\ddot{F}_1)^3}\right) \tag{28}$$

$$u_3 = \frac{Ix}{l}\left(\frac{-\ddddot{F}_3}{g+\ddot{F}_1} + \frac{\ddot{F}_3\ddddot{F}_1}{(g+\ddot{F}_1)^2} + 2\frac{\dddot{F}_3\dddot{F}_1}{(g+\ddot{F}_1)^2} - 2\frac{\ddot{F}_3(\dddot{F}_1)^2}{(g+\ddot{F}_1)^3}\right) \tag{29}$$

The flatness control defined by equation (16), (28) and (29) cannot allow an asymptotic robust tracking of the trajectory in the presence of parameter uncertainties. Hence, a robust correction term will be added to the open control in order to ensure the convergence of the tracking error to zero. This latter constituted of two parts: one part that depends on the tracking error and the second part is a discontinuous term for the robustness which comes from the sliding mode.

IV. Sliding controller based on flatness

Let F_{1d}, F_{2d} , F_{3d} be the reference trajectory for the flat outputs F_1,F_2 and F_3. Let the error dynamic be defined by $e_i = F_i - F_{id}(i = 1, ..., 3)$. The sliding mode control based on flatness law is designed to make sure that the tracking error e_i converges to zero despite the existence of uncertain parameters. The design of the sliding mode control needs two steps: The choice of the sliding surface and the design of the control law. The sliding surfaces for the quadrotor are chosen based on the tracking errors as follows:

$$\sigma_z = \dot{e}_1 + \beta_{11}e_1 \tag{30}$$

$$\sigma_x = \dddot{e}_2 + \beta_{23}\ddot{e}_2 + \beta_{22}\dot{e}_2 + \beta_{21}e_2 \tag{31}$$

$$\sigma_y = \dddot{e}_3 + \beta_{33}\ddot{e}_3 + \beta_{32}\dot{e}_3 + \beta_{31}e_3 \tag{32}$$

where the gains β_{ij}, $(i, j = 1, .., 3)$ can be determined by using pole-placement techniques to ensure that the tracking errors $e_1 = z - z_d$, $e_2 = x - x_d$ and $e_3 = y - y_d$ asymptotically converge to zero. Consider $\sigma = [\sigma_z, \sigma_x, \sigma_y]^T$, the error dynamics restricted to $\sigma = 0$ are defined as follows:

$$\dot{e}_1 + \beta_{11}e_1 = 0 \tag{33}$$

$$\dddot{e}_2 + \beta_{23}\ddot{e}_2 + \beta_{22}\dot{e}_2 + \beta_{21}e_2 = 0 \tag{34}$$

$$\dddot{e}_3 + \beta_{33}\ddot{e}_3 + \beta_{32}\dot{e}_3 + \beta_{31}e_3 = 0 \tag{35}$$

According to [13], to make The sliding surface $\sigma = 0$ attractive. we can force the surface σ to satisfy the dynamics as follows:

$$\dot{\sigma} = -\alpha_i sgn(\sigma) \tag{36}$$

where α_i, $i = 1..3$ represent a positive real constant and sgn is the standard signum function. In order to demonstrate the stability of the error dynamics, let consider the Lyapunov function:

$$V = \frac{1}{2}\dot{\sigma}\sigma \tag{37}$$

the derivate of V is defined as follows:

$$\dot{V} = \sigma\dot{\sigma} \tag{38}$$

Then V is positive and $\dot{V} \leq 0$. Therefore, the asymptotic Lyapunov stability is guaranteed. According to the equations (30), (31), (32) and (36), we obtain:

$$-\alpha_1 sign(\sigma_z) = \ddot{e}_1 + \beta_{11}\dot{e}_1 \tag{39}$$

$$-\alpha_2 sign(\sigma_x) = \ddddot{e}_2 + \beta_{23}\dddot{e}_2 + \beta_{22}\ddot{e}_2 + \beta_{21}\dot{e}_2 \tag{40}$$

$$-\alpha_3 sign(\sigma_y) = \ddddot{e}_3 + \beta_{33}\dddot{e}_3 + \beta_{32}\ddot{e}_3 + \beta_{31}\dot{e}_3 \tag{41}$$

As a consequence, according to equation (39), (40) and (41), we obtain:

$$\ddot{F}_1 = \ddot{F}_{1d} - \beta_{11}\dot{e}_1 - \alpha_1 sign(\sigma_z) \tag{42}$$

$$\ddddot{F}_2 = \ddddot{F}_{2d} - \beta_{23}\dddot{e}_2 - \beta_{22}\ddot{e}_2 - \beta_{21}\dot{e}_2 - \alpha_2 sign(\sigma_x) \tag{43}$$

$$\ddddot{F}_3 = \ddddot{F}_{3d} - \beta_{33}\dddot{e}_3 - \beta_{32}\ddot{e}_3 - \beta_{31}\dot{e}_3 - \alpha_3 sign(\sigma_y) \tag{44}$$

Substituting $\ddot{F}_1$, $\ddddot{F}_2$, $\ddddot{F}_3$ by their new expression defined by equation (42), (43) and (44) in (16), (28) and (29), we obtain the flatness sliding tracking controller for the quadrotor as follows:

$$u_1 = M(g+v_1) \tag{45}$$

$$u_2 = \frac{Iy}{l}\left(\frac{v_2}{g+\ddot{F}_1} - \frac{\ddot{F}_2\ddddot{F}_1}{(g+\ddot{F}_1)^2} - 2\frac{\dddot{F}_2\dddot{F}_1}{(g+\ddot{F}_1)^2} + 2\frac{\ddot{F}_2(\dddot{F}_1)^2}{(g+\ddot{F}_1)^3}\right) \tag{46}$$

$$u_3 = \frac{Ix}{l}\left(\frac{-v_3}{g+\ddot{F}_1} + \frac{\ddot{F}_3\ddddot{F}_1}{(g+\ddot{F}_1)^2} + 2\frac{\dddot{F}_3\dddot{F}_1}{(g+\ddot{F}_1)^2} - 2\frac{\ddot{F}_3(\dddot{F}_1)^2}{(g+\ddot{F}_1)^3}\right) \tag{47}$$

with

$$v_1 = \ddot{F}_{1d} - \beta_{11}\dot{e}_1 - \alpha_1 sign(\sigma_z) \tag{48}$$

$$v_2 = \ddddot{F}_{2d} - \beta_{23}\dddot{e}_2 - \beta_{22}\ddot{e}_2 - \beta_{21}\dot{e}_2 - \alpha_2 sign(\sigma_x) \tag{49}$$

$$v_3 = \ddddot{F}_{3d} - \beta_{33}\dddot{e}_3 - \beta_{32}\ddot{e}_3 - \beta_{31}\dot{e}_3 - \alpha_3 sign(\sigma_y) \tag{50}$$

The sliding controller based on flatness defined by equations (45), (46) and (47) is discontinuous control due the existence of the function $sgn(\sigma)$ which provokes the chattering of the control inputs. Thus, to avoid this problem the function $sgn(\sigma)$ can be replaced by $\frac{\sigma}{\|\ \sigma\|+\tau}$ or tanh(σ), where τ is a tuning parameter utilized to reduce the chattering effect. Figure.2 show the guidance law based on flatness and sliding control applied to the quadrotor.

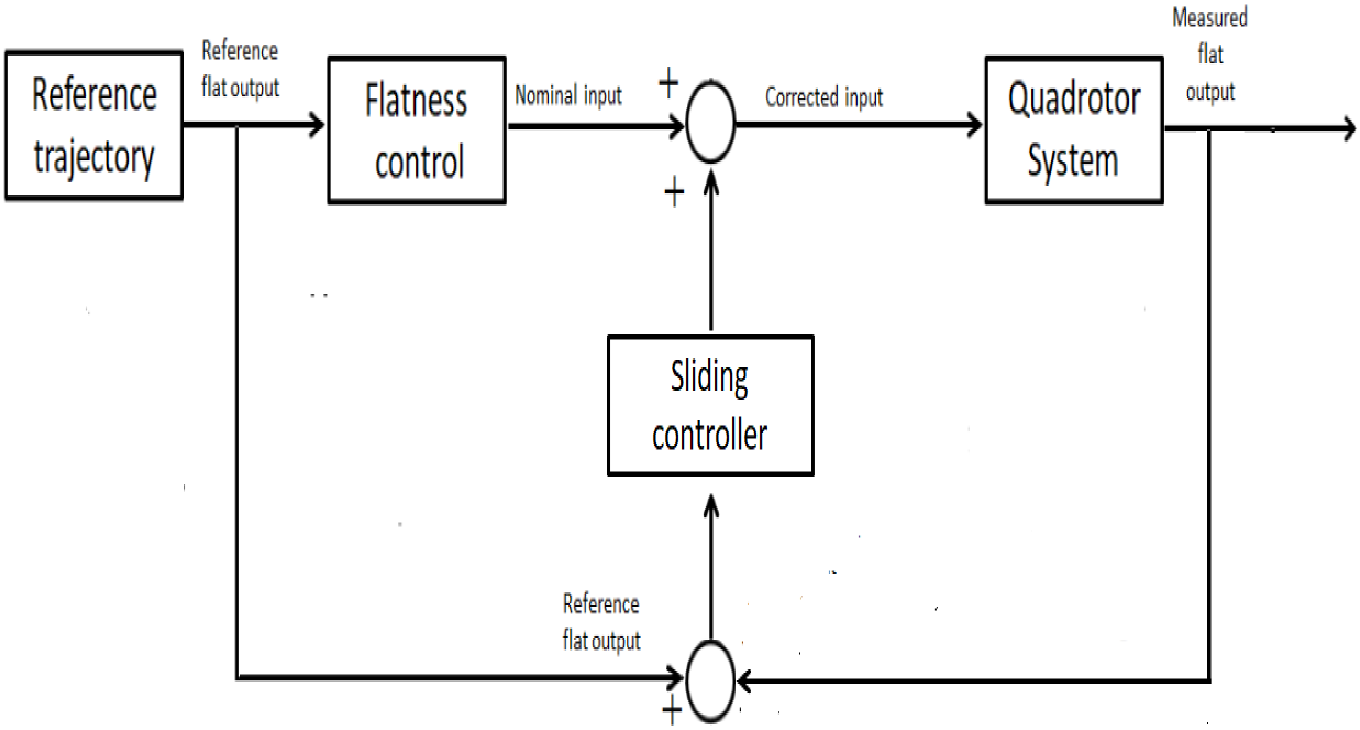


Figure.2 schematic diagram of the guidance law applied to the quadrotor.

V. Simulation and Results

In this section, the proposed sliding control based on flatness for the quadrotor is implemented to illustrate its effectiveness. After that, we consider the AR Drone quadrotor available in laboratories. The AR-Drone parameters were given by [14] and are defined in Table 1.

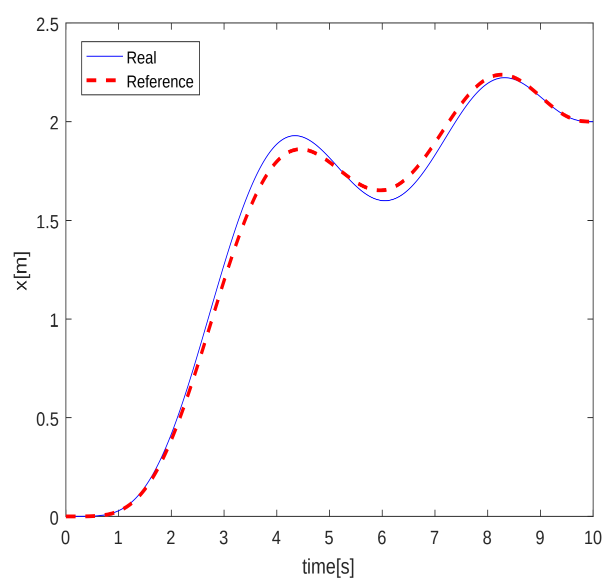
Parameters	Value	Unit
g	9.81	$m.s^{-2}$
M	0.5	Kg
l	0.175	m
I_x	0.002	$Kg.m^2$
I_y	0.002	$Kg.m^2$

Table 1: AR drone parameters

Before dealing with control , it is desired to generate the trajectory which allows the quadrotor to move from an initial state $X(0) = [0,0,0,0,0,0,0,0,0,0]^T$ to a final state $X(10) = [2,0,8,0,0,0,0,0,0,0]^T$. Based on the flatness property of quadrotor system, the initial conditions for the desired trajectories are $F_{1d}(0) = \dot{F}_{1d}(0) = \ddot{F}_{1d}(0) = F_{2d}(0) = \dot{F}_{2d}(0) = \ddot{F}_{2d}(0) = F_{3d}(0) = \dot{F}_{3d}(0) = \ddot{F}_{3d}(0) = 0$ and the final conditions are $F_{1d}(10) = \dot{F}_{1d}(10) = \ddot{F}_{1d}(10) = \dot{F}_{2d}(10) = \ddot{F}_{2d}(10) = \dot{F}_{3d}(10) = \ddot{F}_{3d}(10) = 0, F_{2d}(10) = 2, F_{3d}(10) = 8$. Thereby, any curve that satisfies this condition can be used as a desired trajectory for the quadrotor. In our case, we utilize the Bezier curve of order 8 as a suitable function to approximate the flat output as follows:

$$F_{id}(t) = P_{i0}(1-t)^8 + 8P_{i1}(1-t)^7 t + 28P_{i2}(1-t)^6 t^2 P + 56P_{i3}(1-t)^5 t^3 + 70P_{i4}(1-t)^4 t^4 + 56P_{i5}(1-t)^3 t^5 + 28P_{i6}(1-t)^2 t^6 + 8P_{i7}(1-t)t^7 + P_{i8}t^8$$

where P_{ij} $i = 1..3, j = 0..8$ are the variable parameters. In order to illustrate the efficiency of the sliding flatness-based control, it is considered that the quadrotor undergoes a parametric variation of 20% respectively in M, Ix and Iy. Figure 3, 4 and 5 shows the output response of the system.



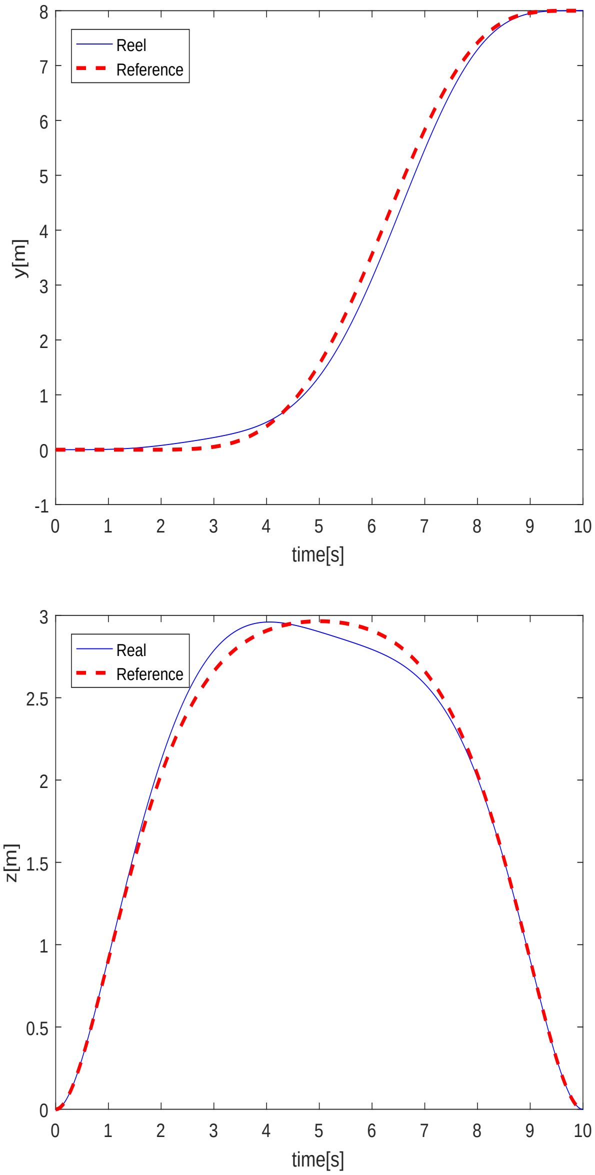
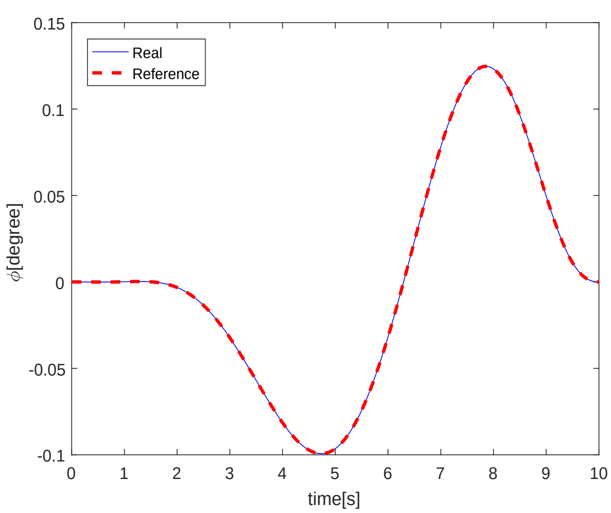


Figure.3 Results for position tracking of quadrotor..



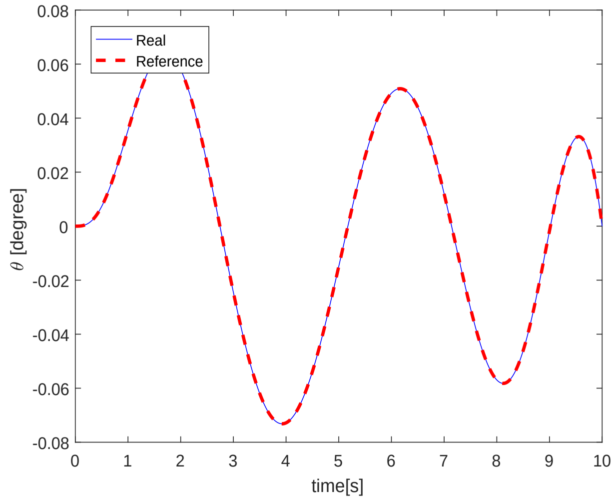


Figure.4 Results for attitude tracking of quadrotor.

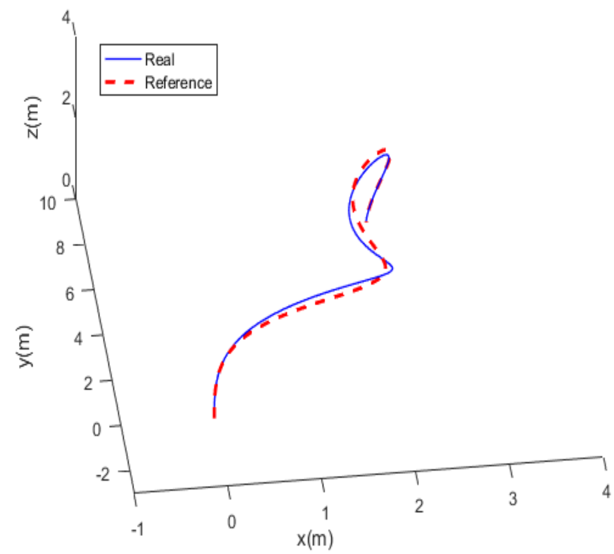
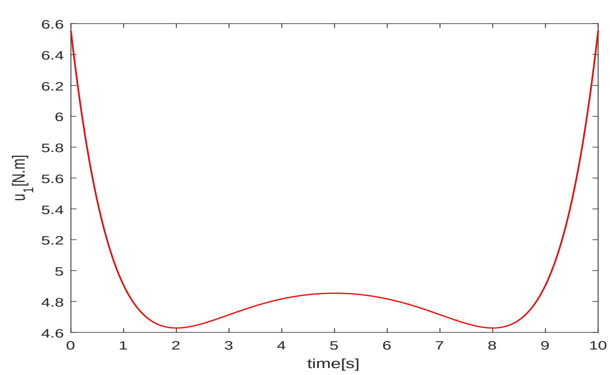


Figure.5 Results for 3D tracking of quadrotor.

It can be observed that the quadrotor can successfully follow the predefined trajectory despite the uncertainties. Hence, the sliding controller based on flatness improves the robustness of the guiding law for the quadrotor. Figure.6 show the sliding controller based on flatness applied to the quadrotor.



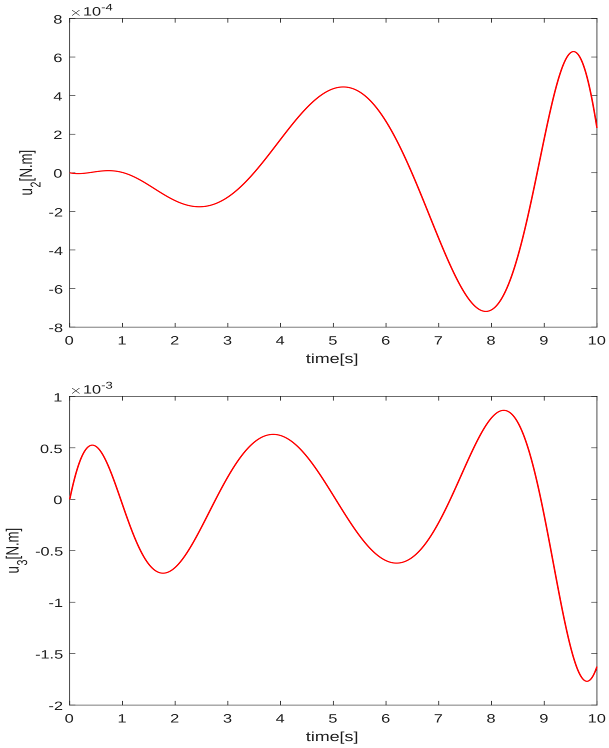


Figure.6 Sliding controller based on flatness

VI. Conclusion

The problem of the robust tracking trajectory for the quadrotor is treated in this paper. Based on the flatness property that the system presents , a sliding mode control is proposed in order to improve the robustness of the tracking scheme. The simulation results indicate that the flatness property with the sliding control is considered as a powerful tool for optimizing planning and robust tracking trajectory. Our future work takes into count the existence of disturbances with unknown bounds in a quadrotor system. Therefore, an adaptive flatness sliding controller will be designed to improve the tracking performance.

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