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Guaranteed Trajectory Tracking Control Based on Interval Observer for Quadrotors

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ABSTRACT
This work proposes guaranteed trajectory tracking control for a quadrotor based on feedforward flatness control and an interval observer. Using the exact feedforward linearization based on the differential flatness property, it is proven that the nonlinear quadrotor model can be transformed into the linear canonical system for which it is easier to create a state feedback controller. Since the quadrotor is subject to bounded uncertainties (parameters, disturbances and noise), the state of this latter cannot be measured properly. Therefore, based on the information of the upper and lower limits of the initial condition, the uncertain parameters, the disturbance and the measurement noise, an interval observer that generates an envelope enclosing every feasible state trajectory is developed. After that, based on the center of the obtained interval observer, flatness feedforward control, combined with estimated feedback control, is proposed to improve the tracking performance of the quadrotor despite the existence of unmeasurable state and bounded uncertainties. The closed-loop stability of the system is proven analytical using the Lyapunov theorem. The numerical simulation is done in order to evaluate the proposed tracking control scheme and the interval observer design.

KEYWORDS
Quadrotor, feedforward flatness, interval observer, uncertain parameter, disturbance, measurement noise.

1. Introduction

In recent years, special attention has been paid to drones because they can successfully follow trajectories and stationary flights. This gives them a lot of practical applications such as military interdiction, transportation and surveillance. In this sense, intensive research efforts have been devoted to the quadrotor helicopters because of their advantages over the conventional drones. This dominance is due to simplicity of the mechanical structure, good maneuverability and low-speed flight. In spite of those advantages, the tracking problem of the quadrotor is still a big challenge because the latter is a highly nonlinear, multivariable, strongly coupled and underactuated system.

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Different nonlinear control methods have been created for the quadrotor, including predictive control (Kocer, 2018), adaptive control (Santos, 2019; Wang, 2017), backstepping control (Chen, 2016), Sliding Mode Control (SMC) (Jeong, 2018; Xiong, 2017) , the immersion and invariance methodology (Zou, 2018), and several other complex strategies. Some reference can be cited related to the latter. In (Jia, 2017), an integral backstepping was combined with SMC in order to create a nonlinear control able to stabilize the quadrotor attitude and to execute the task of trajectory tracking. In (Li, 2014), an adaptive methodology and sliding control were combined in order to design a robust flight control system that would allow the quadrotor to track the reference under the existence of perturbations and uncertainties with unknown bounds. In (Ma, 2016), the author suggested predictive active disturbance rejection control for a quadrotor subjected to disturbance. Thus, predictive control would solve the path tracking problem and the extended state observer was utilized to estimate and compensate the effect of uncertainties and unmodeled dynamics acting on the system.

In this paper, we focus on the differential flatness theory introduced by Fliess (Fliess, 1995), which could facilitate the resolution of trajectory planning and tracking problems for linear and nonlinear systems. With the flatness property, all states and control of the system can be written as a function of the flat outputs and their derivatives. This property allows us to eliminate the utilization of the complex integration process. The concept of differential flatness has been exploited for the conception of feedforward and feedback tracking controllers (Dominic, 2017; Hagenmeyer, 2003; Liu, 2016; Luviano-Juárez, 2015), for a nonlinear system. Flatness has specific advantages when used in nonlinear control systems consisting in transforming the latter into a linearized Burnovsky form, for which the development of a feedback controller will be easier. In addition, due to the structure of the differential parameterization obtained by flatness, nonlinear effects as well as non-modeled dynamics can be defined as disturbances.

In the last decade, the flatness property has been explored in motion planning and trajectory tracking for a quadrotor. In (Chamseddine, 2012), the author developed a trajectory planning/replanning strategy based on the Bezier polynomials and the flatness property for a quadrotor in order to drive the system from an initial position to a final position as fast as possible under actuator faults and without hitting system constraints. In (Limaverde, 2016), the flatness theory, combined with predictive control, was utilized to ensure an online trajectory tracking. In (Taamallaha, 2017), the author presented a flight control system composed of optimal planning based on differential flatness, and on robust tracking control for the quadrotor. In (Lu, 2016), the author put forward an online optimization process based on the differential flatness to generate the desired reference and a backstepping controller to track the obtained optimal trajectory.

Although those control algorithms can improve the trajectory tracking performance when applied to the quadrotor, all of them are designed based on the full-state feedback. Usually, due to space limitation, technical problems or immense costs, it is not possible to measure all the state variables of the quadrotor. Hence, it is necessary to use an observer to estimate the quadrotor state. Generally, the un-measurable state is not the only problem that can disrupt the flight operation of the quadrotor because most of flatness tracking controllers and observers applied to this latter are based on the assumption that the system parameters are constant and the disturbance and measurement noise is negligible. In real applications, some parameters can be poorly defined or their value changes over time. Furthermore, it is important to consider the impact of noise and disturbances on the quadrotor system in order to have guaranteed results. Consequently, the observer design for the quadrotor represents a big challenge

under the existence of uncertain parameters, disturbances and noise measurements.
In the last decade, interval observers have become a robust approach to dealing with accurate state estimation problems in the presence of all perturbations (disturbances, unknown parameters and measurement noise). Starting from the knowledge of the upper and lower limits of the initial conditions of the system state, the parameter uncertainties, the disturbance and the noise measurement, interval observers can be developed to produce upper and lower bounds of state variables of dynamical systems at every time instant. Accordingly, the bounds offer intervals where the estimated variables are sure to stay for transient periods during which the classical observers cannot ensure any guarantee. In the literature, the observer interval technique has been used in several applications (Ifqir, 2017; Lamouchi, 2017a; Meslem, 2011; Zhang, 2017) and has been particularly successfully utilized in the field of biological processes (Alcaraz-González, 2007; Bernard, 2004; Goffaux, 2009; Gouzé, 2000). In (Efimov, 2016), a survey about an interval observer and its application was suggested. One of the important conditions to create interval observers treated the cooperativity of the estimation error dynamics, that was relaxed in (Mazenc, 2010; Raïssi, 2012). In those work, it has been demonstrated that according to certain conditions applying similarity transformation, a Hurwitz matrix could be transformed to a Metzler and Hurwitz one (cooperative). The interval observer design for nonlinear systems with model uncertainty was discussed in (Efimov, 2013; Meyer, 2017; Wang, 2015; Zheng, 2016a). However, The existing results showed that the observation error would converge to an interval whose size depended on the value of uncertainty. In addition, any state belongs to this interval can be considered as a guaranteed state estimation. Therefore, compared to punctual observer, the estimation interval guaranteed more robustness when dealing with bounded uncertainties. This property encourages us to exploit the advantage of interval observer in practical application such as tracking trajectory for quadrotor. Because, in real application, this latter is subject to bounded uncertainties (parameters, disturbances and noise).
In this paper, our contribution consists in the design of a guaranteed trajectory tracking control for the quadrotor despite the existence of bounded uncertainties. Based on feedforward flatness control, it is possible to change the quadrotor equation model into a linear canonical (Brunovsky) form. For the obtained linearized system, it is simpler to develop a state feedback controller employing the method for linear feedback controller synthesis. To improve the tracking robustness of the quadrotor subjected to bounded uncertainties, an interval observer is developed to create an envelope containing all possible state estimations based on the upper and lower values of initial conditions, uncertain parameters, disturbances and noise. Subsequently, the center of the interval observer is considered as a robust state estimation of the quadrotor. Finally, based on this robust estimation, flatness feedforward control, combined with an estimated feedback law, is developed in order to guarantee that the quadrotor tracks the reference trajectory in a precise interval.
This document is organized as follows. In section II, we present the quadrotor model. In section III, the tracking control based on the flatness feedforward control is defined. In section IV, the design of an interval observer for the quadrotor is defined. Section V is devoted to the design of guaranteed tracking control based on the interval observer. Section VI deals with the simulation results, and section VII concludes the paper.

2. Quadrotor model

A quadrotor (Figure 1) is an aircraft with four engines installed on a cross usually made of carbon fiber. The front and rear engines rotate clockwise while the right and left engines rotate in the opposite direction. Compared with other types of drones, the quadrotor has precise characteristics that permit the performance of difficult or impossible applications. In order to establish the dynamic model of a quadrotor, two frames are necessary to be defined, the first is the base reference frame $E : \{O, e_x, e_y, e_z\}$ which is fixed to the ground, the second is the body-fixed reference frame $B : \{O_q, x, y, z\}$ whose origin coincides with the center of the quadrotor structure. The transformation matrix R_t between the base and the body-fixed frame is defined as follows:

$$R_t = \begin{bmatrix} cos(\theta)cos(\phi) & cos(\psi)sin(\theta)sin(\phi) - sin(\psi)cos(\phi) & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ cos(\theta)sin(\phi) & sin(\psi)sin(\theta)sin(\phi) + cos(\psi)cos(\phi) & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi \\ -sin(\theta) & sin(\phi)cos(\theta) & cos(\theta)cos(\phi) \end{bmatrix} \quad (1)$$

where θ, ϕ and ψ are the Euler angles.
There are various models that can be utilized to define the dynamic equations of a quadrotor depending on the assumptions and simplifications made to the models. For example, taking the aerodynamics of the system into consideration or neglecting it will make a big difference in the dynamic equations.

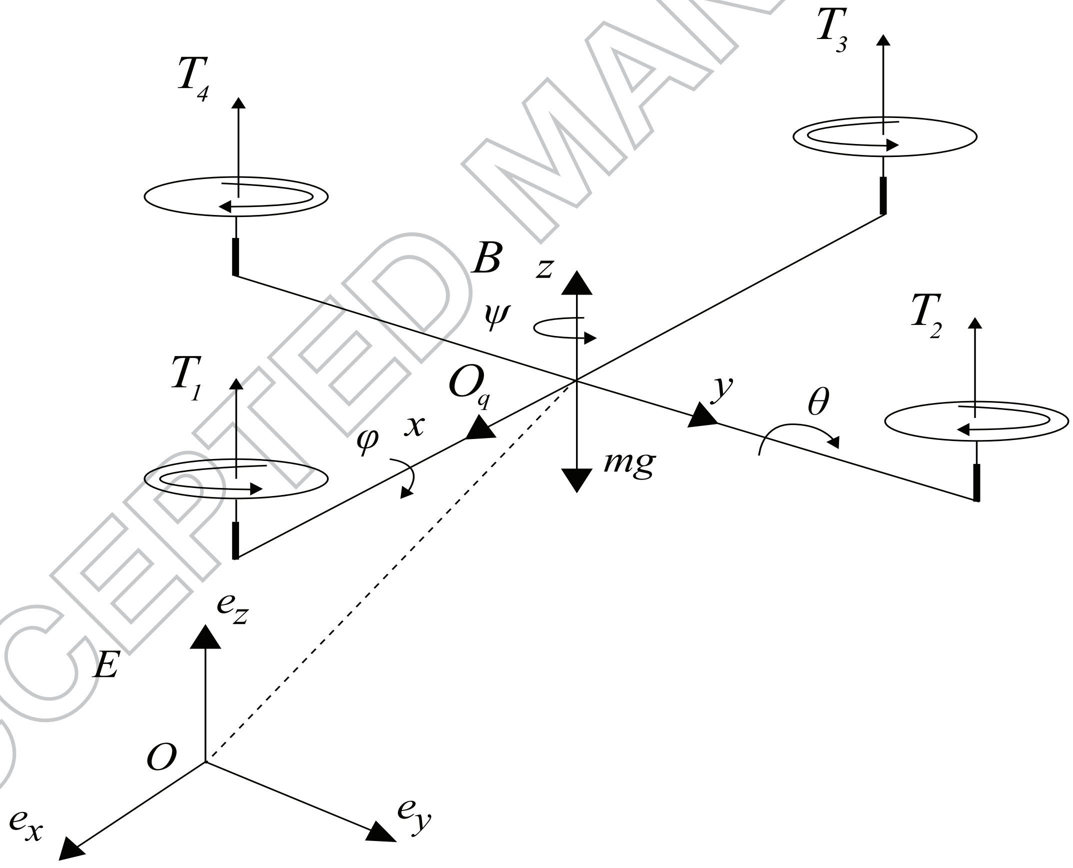

Figure 1. Quadrotor aircraft scheme.

According to Newton-Euler equations, the transnational dynamic equation can be obtained as follows:

$$m\ddot{X}_p + mge_3 = R_t u_1 e_3 \qquad (2)$$

where m represents the quadrotor mass, g represents the accelerant of gravity, $Xp = [x, y, z]^T$ is the quadrotor position, $e_3 = [0, 0, 1]^T$ is the unit vector along the z axis of the inertial reference frame, $u_1 = \sum_{i=1}^{4}(T_i)$ is the total translational force, and T_i $(i = 1, 2, 3, 4)$ are thrusts generated by four rotors and can be considered as the real control inputs to the system. The translational dynamics of the quadrotor defined by equation (2) can be written as follows:

$$\begin{aligned} \ddot{x} &= \frac{\sum_{i=1}^{4}(T_i)}{m}(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) \\ \ddot{y} &= \frac{\sum_{i=1}^{4}(T_i)}{m}(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) \\ \ddot{z} &= \frac{\sum_{i=1}^{4}(T_i)}{m}(\cos\theta\cos\phi) - g \end{aligned} \qquad (3)$$

The rotational dynamics of the quadrotor is defined as follows:

$$I\dot{\Omega} = -\Omega \times I\Omega + \tau_f \qquad (4)$$

where $\Omega = [p, q, r]^T$ represents the angular rate, $I = diag[I_x, I_y, I_z]$ represents the inertia matrix of the quadrotor, I_x, I_y and I_z are the moments of inertia along the directions x, y and z, and $\tau_f = [u_2, u_3, u_4]^T$ is the total control torque for rotational motion. According to (Zheng, 2014), the rotational dynamics of the quadrotor defined by (4) can be written as follows:

$$\begin{aligned} \ddot{\theta} &= \dot{\phi}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{l}{I_y}(T_1 - T_3) \\ \ddot{\phi} &= \dot{\theta}\dot{\psi}(\frac{I_z - I_y}{I_x}) + \frac{l}{I_x}(-T_2 + T_4) \\ \ddot{\psi} &= \dot{\phi}\dot{\theta}(\frac{I_y - I_x}{I_z}) + \frac{c}{I_z}(T_1 - T_2 + T_3 - T_4) \end{aligned} \qquad (5)$$

where c is the force of the moment scaling factor, and l is the distance from the center of gravity to each rotor.

To simplify the presentation of the quadrotor system, the virtual control variables u_1, u_2, u_3 and u_4 are expressed as a function of the lift forces T_1, T_2, T_3 and T_4 as follows:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \qquad (6)$$

where u_1 represents a total thrust on the body in the z-axis, u_2 and u_3 are the pitch and roll inputs, and u_4 is the yawing moment input. From equations (3),

(5) and (6), the mathematical model of the quadrotor flight system can be rewritten as:

$$
\begin{aligned}
\ddot{x} &= \frac{u_1}{m}(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) \\
\ddot{y} &= \frac{u_1}{m}(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) \\
\ddot{z} &= \frac{u_1}{m}(\cos\theta\cos\phi) - g \\
\ddot{\theta} &= \dot{\phi}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{l}{I_y}u_2 \\
\ddot{\phi} &= \dot{\theta}\dot{\psi}(\frac{I_z - I_y}{I_x}) + \frac{l}{I_x}u_3 \\
\ddot{\psi} &= \dot{\phi}\dot{\theta}(\frac{I_y - I_x}{I_z}) + \frac{c}{I_z}u_4
\end{aligned}
\tag{7}
$$

where the state of the quadrotor system is $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}] = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \theta, \dot{\theta}, \phi, \dot{\phi}, \psi, \dot{\psi}]^T$, and the control input is $U = [u_1, u_2, u_3, u_4]^T$.

3. Tracking control

In this section, flatness-based feedforward control combined with feedback law will be created for the quadrotor system (7) in order to track the reference trajectories. This concept was developed by (Hagenmeyer, 2003), and applied successfully for mobile robots (Luviano-Juárez, 2015) and helicopters (Formentin, 2011). The following nonlinear system:

$$\dot{x} = f(x, u), x \in \mathbb{R}^n, u \in \mathbb{R}^m \tag{8}$$

is differentially flat if we find the following outputs:

$$F = \xi(x, u, \dot{u}, ..., u^{(r-1)}) \tag{9}$$

and

$$x = \gamma(F, \dot{F}, \ddot{F}...., F^{(\alpha)}) \tag{10}$$

$$u = \gamma(F, \dot{F}, \ddot{F}..., F^{(\alpha+1)}) \tag{11}$$

where α and r are finite multi-indices, and ξ and γ are smooth vector functions of the output vector F.
Roughly speaking, a flat system is one whose state and control variables can be written as a function of these flat outputs and its derivatives. For a differentially flat system, when the desired trajectory F_d is known, the desired state x_d and the feedforward control u_d can be defined as follows:

$$x_d = \gamma(F_d, \dot{F}_d, \ddot{F}_d..., F_d^{(\alpha)}) \tag{12}$$

$$u_d = \gamma(F_d, \dot{F}_d, \ddot{F}_d..., F_d^{(\alpha+1)}) \tag{13}$$

The flatness-based-open-loop control (13) is known as an exact feedforward linearization, because it gives a Brunovsky representation during all the times if the initial conditions of the desired trajectory and the system are coherent. Moreover, a feedback control v is included in the flatness feedforward control (13) in order to enhance the tracking performance. Therefore, a Flatness Based Tracking Control (FTC) is obtained. This latter is two degrees of freedom which includes two parts: a feedforward part, $u_d = \gamma(F_d, \dot{F}_d, \ddot{F}_d..., F_d^{(\alpha+1)})$, and a feedback part v which represents simple linear control able to stabilize the obtained linearized system. However, FTC is given as follows:

$$u_{FTC} = \gamma(F_d, \dot{F}_d, \ddot{F}_d..., v) \tag{14}$$

with

$$v = F_d^{(\alpha+1)} - \sum_{i=1}^{\alpha+1}(K_i e_{ri}) \tag{15}$$

where $e_{ri} = F_i - F_{di}$, $i = 1, 2, 3, ..., \alpha + 1$ and K_i $i = 1...\alpha + 1$ are the feedback gains that are calculated by using pole-placement techniques to ensure the convergence of the tracking error to zero.

It can be shown that the quadrotor model is a differentially flat system whose flat outputs are given by $F = [F_1, F_2, F_3, F_4, F_5, F_6]^T = [x, y, z, \theta, \phi, \psi]^T$. Hence, the quadrotor state can be defined as follows:

$$\begin{cases} x_1 = \begin{bmatrix}1\,0\,0\,0\,0\,0\end{bmatrix} F, x_2 = \begin{bmatrix}1\,0\,0\,0\,0\,0\end{bmatrix} \dot{F} \\ x_3 = \begin{bmatrix}0\,1\,0\,0\,0\,0\end{bmatrix} F, x_4 = \begin{bmatrix}0\,1\,0\,0\,0\,0\end{bmatrix} \dot{F} \\ x_5 = \begin{bmatrix}0\,0\,1\,0\,0\,0\end{bmatrix} F, x_6 = \begin{bmatrix}0\,0\,1\,0\,0\,0\end{bmatrix} \dot{F} \\ x_7 = \begin{bmatrix}0\,0\,0\,1\,0\,0\end{bmatrix} F, x_8 = \begin{bmatrix}0\,0\,0\,1\,0\,0\end{bmatrix} \dot{F} \\ x_9 = \begin{bmatrix}0\,0\,0\,0\,1\,0\end{bmatrix} F, x_{10} = \begin{bmatrix}0\,0\,0\,0\,1\,0\end{bmatrix} \dot{F} \\ x_{11} = \begin{bmatrix}0\,0\,0\,0\,0\,1\end{bmatrix} F, x_{12} = \begin{bmatrix}0\,0\,0\,0\,0\,1\end{bmatrix} \dot{F} \end{cases} \tag{16}$$

Therefore, it appears that the quadrotor system is differential flat. Using equations (7), the control inputs u_1, u_2, u_3 and u_4 can be written as a function of the flat outputs F and their derivatives as follows:

$$u_1 = m\sqrt{(\ddot{F}_1)^2 + (\ddot{F}_2)^2 + (\ddot{F}_3 + g)^2} \tag{17}$$

$$u_2 = \frac{I_y}{l}\ddot{F}_4 - (\dot{F}_5\dot{F}_6)\frac{I_z - I_x}{l} \tag{18}$$

$$u_3 = \frac{I_x}{l}\ddot{F}_5 - (\dot{F}_4\dot{F}_6)\frac{I_z - I_y}{l} \tag{19}$$

$$u_4 = \frac{I_z}{c}\ddot{F}_6 - (\dot{F}_4\dot{F}_5)\frac{I_y - I_x}{c} \tag{20}$$

Let F_{1d}, F_{2d}, F_{3d}, F_{4d}, F_{5d} and F_{6d} be the desired reference trajectories of the flat output F_1, F_2, F_3, F_4, F_5 and F_6. Based on the desired reference trajectories, the flatness feedforward control can be obtained when replacing the flat output F and their derivative by the desired flat output F_d and their derivatives in the control inputs u_1, u_2, u_3 and u_4 as follows:

$$u_{1d} = m\sqrt{(\ddot{F}_{1d})^2 + (\ddot{F}_{2d})^2 + (\ddot{F}_{3d} + g)^2} \tag{21}$$

$$u_{2d} = \frac{Iy}{l}\ddot{F}_{4d} - (\dot{F}_{5d}\dot{F}_{6d})\frac{I_z - I_x}{l} \tag{22}$$

$$u_{3d} = \frac{Ix}{l}\ddot{F}_{5d} - (\dot{F}_{4d}\dot{F}_{6d})\frac{I_z - I_y}{l} \tag{23}$$

$$u_{4d} = \frac{I_z}{c}\ddot{F}_{6d} - (\dot{F}_{4d}\dot{F}_{5d})\frac{I_y - I_x}{c} \tag{24}$$

Now, in order to ensure good tracking of the desired trajectory, the feedback controller can be constructed as follows:

$$v_i = \ddot{F}_{id} - k_{\alpha i}\dot{e}_{ri} - k_{\beta i}e_{ri} \quad (i = 1,2,3,4,5,6) \tag{25}$$

with $e_{ri} = F_i - F_{id}$ $i = 1,2,3,4,5,6$.
According to the pole placement technique, the gains $k_{\beta i}$ and $k_{\alpha i}$ (i=1,2,3,4,5,6) can be chosen so that the characteristic polynomial ($s^2 + K_{\alpha i}s + K_{\beta i}$) is Hurwitz, which means that the two poles of the latter polynomial can be placed at - ω_{ci} as follows:

$$s^2 + K_{\alpha i}s + K_{\beta i} = s^2 + 2\xi_i\omega_{ic} + \omega_{ic}^2 \quad i = 1,2,3,4,5,6. \tag{26}$$

where the parameters ξ_i $(i = 1,2,3,4,5,6)$ are the damping coefficient, and ω_{ic} $(i = 1,2,3,4,5,6)$ are the bandwidths of the controller. When considering the response time and the overshoot in selecting the gain, the critical damping of the second-order system whose damping coefficient is 1 can be chosen to balance the response time and the overshoot performance. Then, based on equation (26), it is easy to follow that the controller gain can be computed as follows:

$$K_{\beta i} = \omega_{ic}^2, K_{\alpha i} = 2\omega_{ic} \quad i = 1,2,3,4,5,6. \tag{27}$$

When replacing $\ddot{F}_{id}$ (i=1,2,3,4,5,6) by the feedback control v_i (i=1,2,3,4,5,6) in u_{1d}, u_{2d}, u_{3d} and u_{4d}, the FTC applied the quadrotor can be obtained as follows:

$$u_{FTC1} = m\sqrt{(\ddot{F}_{1d} - k_{\alpha 1}\dot{e}_{r1} - k_{\beta 1}e_{r1})^2 + (\ddot{F}_{2d} - k_{\alpha 2}\dot{e}_{r2} - k_{\beta 2}e_{r2})^2 + (\ddot{F}_{3d} - k_{\alpha 3}\dot{e}_{r3} - k_{\beta 3}e_{r3} + g)^2} \tag{28}$$

$$u_{FTC2} = \frac{Iy}{l}(\ddot{F}_{4d} - k_{\alpha 4}\dot{e}_{r4} - k_{\beta 4}e_{r4}) - (\dot{F}_{5d}\dot{F}_{6d})\frac{I_z - I_x}{l} \tag{29}$$

$$u_{FTC3} = \frac{Ix}{l}(\ddot{F}_{5d} - k_{\alpha 5}\dot{e}_{r5} - k_{\beta 5}e_{r5}) - (\dot{F}_{4d}\dot{F}_{6d})\frac{I_z - I_y}{l} \tag{30}$$

$$u_{FTC4} = \frac{I_z}{c}(\ddot{F}_{6d} - k_{\alpha 6}\dot{e}_{r6} - k_{\beta 6}e_{r6}) - (\dot{F}_{4d}\dot{F}_{5d})\frac{I_y - I_x}{c} \tag{31}$$

When applying the FTC defined by equations (28-31) to the quadrotor system (7), the dynamics of the closed-loop tracking error is defined as follows:

$$\dot{e}_r = H_r e_r \tag{32}$$

with
$e_r = [e_{r1}, \dot{e}_{r1}, e_{r2}, \dot{e}_{r2}, e_{r3}, \dot{e}_{r3}, e_{r4}, \dot{e}_{r4}, e_{r5}, \dot{e}_{r5}, e_{r6}, \dot{e}_{r6}]^T$

$$H_r = \begin{bmatrix} H_1 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & H_2 & 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & H_3 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & H_4 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 & H_5 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & H_6 \end{bmatrix}, 0_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, H_1 = \begin{bmatrix} 0 & 1 \\ -k_{\beta 1} & -k_{\alpha 1} \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0 & 1 \\ -k_{\beta 2} & -k_{\alpha 2} \end{bmatrix} H_3 = \begin{bmatrix} 0 & 1 \\ -k_{\beta 3} & -k_{\alpha 3} \end{bmatrix}, H_4 = \begin{bmatrix} 0 & 1 \\ -k_{\beta 4} & -k_{\alpha 4} \end{bmatrix},$$

$$H_5 = \begin{bmatrix} 0 & 1 \\ -k_{\beta 5} & -k_{\alpha 5} \end{bmatrix}, H_6 = \begin{bmatrix} 0 & 1 \\ -k_{\beta 6} & -k_{\alpha 6} \end{bmatrix}$$

The closed-loop stability of the error tracking system (32) can be ensured by appropriately choosing the controller poles. FTC ensures a good tracking of reference trajectories for the quadrotor provided that the parameters are exact, the states are all measurable and the measurement noise and the disturbance are negligible, which is not really practical. For this reason, the quadrotor model defined in (7) does not really describe the unpredictable changes in the flight environment, and consequently the performance of the tracking control can be degraded. Hence, it is important to design a robust observer and control for a new uncertain quadrotor model that considers the existence of bounded uncertain parameters, disturbances and noise.

4. Guaranteed interval observer

In this section, an interval observer is applied for an uncertain quadrotor, whose mathematical model is defined as follows:

$$\begin{cases} \ddot{x} = \frac{u_{FTC1}}{m}(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) + d_x \\ \ddot{y} = \frac{u_{FTC1}}{m}(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi) + d_y \\ \ddot{z} = \frac{u_{FTC1}}{m}(\cos\theta \cos\phi) - g + d_y \\ \ddot{\theta} = \dot{\phi}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{l}{I_y} u_{FTC2} + d_\theta \\ \ddot{\phi} = \dot{\theta}\dot{\psi}(\frac{I_z - I_y}{I_x}) + \frac{l}{I_x} u_{FTC3} + d_\phi \\ \ddot{\psi} = \dot{\phi}\dot{\theta}(\frac{I_y - I_x}{I_z}) + \frac{c}{I_z} u_{FTC4} + d_\psi \\ Y = [x, y, z, \theta, \phi, \psi]^T + [\eta_x, \eta_y, \eta_z, \eta_\theta, \eta_\phi, \eta_\psi]^T \end{cases} \tag{33}$$

In a matrix form, the model is as follows:

$$\begin{cases} \dot{X} = AX + \Gamma(X, U_{FTC}) + d \\ Y = CX + \eta \end{cases} \tag{34}$$

with
$X = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \theta, \dot{\theta}, \phi, \dot{\phi}, \psi, \dot{\psi}]^T$, $U_{FTC} = [u_{FTC1}, u_{FTC2}, u_{FTC3}, u_{FTC4}]^T$, $d = [0, d_x, 0, d_y, 0, d_z, 0, d_\theta, 0, d_\phi, 0, d_\psi]^T$, $\eta = [\eta_x, \eta_y, \eta_z, \eta_\theta, \eta_\phi, \eta_\psi]^T$.

$$A = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}, C = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix},$$

$$\Gamma(X, U_{FTC}) = \begin{bmatrix} 0 \\ \frac{u_{FTC1}}{m}(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) \\ 0 \\ \frac{u_{FTC1}}{m}(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi) \\ 0 \\ \frac{u_{FTC1}}{m}(\cos\theta \cos\phi) - g \\ 0 \\ \dot{\phi}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{l}{I_y} u_{FTC2} \\ 0 \\ \dot{\theta}\dot{\psi}(\frac{I_z - I_y}{I_x}) + \frac{l}{I_x} u_{FTC3} \\ 0 \\ \dot{\phi}\dot{\theta}(\frac{I_y - I_x}{I_z}) + \frac{c}{I_z} u_{FTC4} \end{bmatrix}$$

Mass m, the distance from the center of gravity to each rotor l, the moments of inertia I_x, I_y and I_z and the force of the moment scaling factor c represent the uncertain parameters of the quadrotor model, d represent a sinusoidal wind disturbance affecting the quadrotor, η represent the measurement noise. $\Gamma(X, U_{FTC})$ represents the nonlinear term with the appropriate dimension, and $U_{FTC} \in \mho$ is a bounded input where $\mho$ is a compact set.

The main idea of designing an interval observer of the uncertain system (34) is to create lower and upper bound, $\underline{X}(t)$ and $\overline{X}(t)$, of the real state $X(t)$, which allows us to guarantee that this latter belongs to a specific interval. The observer interval design of an uncertain nonlinear system (34) requires the following assumptions:

Assumptions 1. Pair (A, C) is observable or at least detectable.

Assumptions 2. There exists gain L such that matrix $(A - LC)$ is Hurwitz and Metzler (off-diagonal elements are positive).

Assumptions 3. All the uncertainties (parameters, disturbance, noise) are unknown but bounded with known bound $\underline{m}$, $\overline{m}$, $\underline{l}$, $\overline{l}$, $\underline{I}_x$, $\overline{I}_x$, $\underline{I}_y$, $\overline{I}_y$, $\underline{I}_z$, $\overline{I}_z$, $\underline{c}$, $\overline{c}$, $\underline{d}$, $\overline{d}$, $\underline{\eta}$ and $\overline{\eta}$.

Assumptions 4.(Zheng, 2016b) Assume that there exist two locally Lipschitz continuous functions $\underline{\Gamma}(\underline{X}, \overline{X}, U_{FTC})$ and $\overline{\Gamma}(\underline{X}, \overline{X}, U_{FTC})$ satisfying:

$$\begin{cases} \underline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) \leq \Gamma(X, U_{FTC}) \leq \overline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) \\ for\ \ \underline{X} \leq X \leq \overline{X}\ \ and\ \forall U_{FTC} \in \mho \subset \mathbb{R}^m, Y \in \mathbb{R}^k \end{cases} \tag{35}$$

and for a given submultiplicative norm $||.||$, equation (36) and (37) are as follows:

$$||\overline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) - \Gamma(X, U_{FTC})|| \leq \gamma_1||\overline{X} - X|| + \gamma_2||\underline{X} - X|| + \gamma_3 \tag{36}$$

$$||\underline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) - \Gamma(X, U_{FTC})|| \leq \gamma_4||\overline{X} - X|| + \gamma_5||\underline{X} - X|| + \gamma_6 \tag{37}$$

where γ_i $i = 1, 2, 3, 4, 5, 6$ are positive constants. According to assumptions $1, 2, 3$ and 4 , the interval observer of system (34) is defined as follows:

$$\begin{cases} \dot{\overline{X}} = A\overline{X} + \overline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) + \overline{d} + LC(X - \overline{X}) + L\overline{\eta} \\ \dot{\underline{X}} = A\underline{X} + \underline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) + \underline{d} + LC(X - \underline{X}) + L\underline{\eta} \\ \underline{X}(0) \leq X(0) \leq \overline{X}(0) \end{cases} \tag{38}$$

Consider the upper observer error $\overline{e}_{ob} = \overline{X} - X$ and the lower observer error $\underline{e}_{ob} = X - \underline{X}$ whose dynamics are defined as follows:

$$\begin{cases} \dot{\overline{e}}_{ob} = H_{ob}\overline{e}_{ob} + \overline{\Pi}(\underline{X}, \overline{X}, U_{FTC}) \\ \dot{\underline{e}}_{ob} = H_{ob}\underline{e}_{ob} + \underline{\Pi}(\underline{X}, \overline{X}, U_{FTC}) \end{cases} \tag{39}$$

with $H_{ob} = A - LC, \overline{\Pi} = \overline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) - \Gamma(X, U_{FTC}) + \overline{d} - d + L(\overline{\eta} - \eta)$ and $\underline{\Pi} = \Gamma(X, U_{FTC}) - \underline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) + d - \underline{d} + L(\eta - \underline{\eta})$.
Functions $\overline{\Pi}$ and $\underline{\Pi}$ are globally Lipschitz, and consequently for $\underline{X} \leq X \leq \overline{X}$ and for a chosen submultiplicative norm, there exist positive constants π_i $i = 1, 2, 3, 4, 5, 6$ such that:

$$||\overline{\Pi}(\underline{X}, \overline{X}, U_{FTC})|| \leq \pi_1||\overline{X} - X|| + \pi_2||\underline{X} - X|| + \pi_3 \tag{40}$$

$$||\underline{\Pi}(\underline{X}, \overline{X}, U_{FTC})|| \leq \pi_4||\overline{X} - X|| + \pi_5||\underline{X} - X|| + \pi_6 \tag{41}$$

Since H_{ob} is assumed to be non-negative, $\overline{\Pi}$ and $\underline{\Pi}$ are non-negative. Then for any initial condition $\underline{X}(0)$ and $\overline{X}(0)$ chosen such that $\underline{e}_{ob}(0)$ and $\overline{e}_{ob}(0)$ are non-negative, the dynamics of the interval estimation errors $\overline{e}_{ob}$ and $\underline{e}_{ob}$ stay always nonnegative for all time t, so the state is bounded as follows:

$$\underline{X}(t) \leq X(t) \leq \overline{X}(t) \quad \forall t \geq 0 \tag{42}$$

For the quadrotor system (34), it is impossible to compute gain L such that $(A - LC)$ is Metzler and Hurwitz, which presents an obstacle for the construction of the interval observer. This drawback was surmounted via a change of coordinates $Z = GX$ such that $E = GH_{ob}G^{-1}$ was Metzler and Hurwitz. The detail of calculating the transformation matrix G can be found in (Raïssi, 2012).

Lemma 4.1. *(Raïssi, 2012) Consider that there exist a gain L such that matrix $A - LC$ and a Metzler matrix M_T possess the same eigenvalues. If there exist two vectors e_1 and e_2 such that the pairs $(A - LC, e_1)$ and $(M_T, e_2)$are observable, then we get:*

$$G = S_2^{-1} * S_1 \tag{43}$$

with

$$S_1 = \begin{bmatrix} e_1 \\ . \\ . \\ e_1(A-LC)^{n-1} \end{bmatrix}, S_2 = \begin{bmatrix} e_2 \\ . \\ . \\ e_2 M_T \end{bmatrix} \tag{44}$$

Actually, when introducing the new variables $Z = GX$, system (34) can be presented as follows:

$$\begin{cases} \dot{Z} = GAG^{-1}Z + G\Gamma(G^{-1}Z, U_{FTC}) + Gd \\ Y = CG^{-1}Z + \eta \end{cases} \tag{45}$$

The interval observer of the new system (45) is defined as follows:

$$\begin{cases} \dot{\overline{Z}} = E\overline{Z} + G^+(\overline{\Gamma}(\underline{Z}, \overline{Z}, U_{FTC}) + \overline{d}) - G^-(\underline{\Gamma}(\underline{Z}, \overline{Z}, U_{FTC}) + \underline{d}) + GLY + GL\overline{\eta} \\ \dot{\underline{Z}} = E\underline{Z} + G^+(\underline{\Gamma}(\underline{Z}, \overline{Z}, U_{FTC}) + \underline{d}) - G^-(\overline{\Gamma}(\underline{Z}, \overline{Z}, U_{FTC}) + \overline{d}) + GLY + GL\underline{\eta} \\ \overline{Z}(0) = G^+\overline{X}(0) - G^-\underline{X}(0) \\ \underline{Z}(0) = G^+\underline{X}(0) - G^-\overline{X}(0) \end{cases} \tag{46}$$

with $E = G(A - LC)G^{-1}, G^+ = max(0, G), G^- = G^+ - G$.
After designing the interval observer of the new system (45), the upper and lower states of the original system can be deduced as follows:

$$\begin{cases} \dot{\overline{X}} = R^+\overline{Z} - R^-\underline{Z} \\ \dot{\underline{X}} = R^+\underline{Z} - R^-\overline{Z} \\ \underline{X}(0) \leq X(0) \leq \overline{X}(0) \end{cases} \tag{47}$$

with $R = G^{-1}, R^+ = max(0, R), R^- = R^+ - R$.
Consider the error equations $\overline{e}_z = \overline{Z} - Z$ and $\underline{e}_z = Z - \underline{Z}$. The dynamics of upper and lower observer errors are defined as follows:

$$\begin{cases} \dot{\overline{e}}_z = E\overline{e}_z + \overline{\Pi}_z(\underline{Z}, \overline{Z}, U_{FTC}) \\ \dot{\underline{e}}_z = E\underline{e}_z + \underline{\Pi}_z(\underline{Z}, \overline{Z}, U_{FTC}) \end{cases} \tag{48}$$

with $\overline{\Pi}_z = G^+(\overline{\Gamma}(\underline{Z}, \overline{Z}, U_{FTC}) + \overline{d}) - G^-(\underline{\Gamma}(\underline{Z}, \overline{Z}, U_{FTC}) + \underline{d}) - G\Gamma(G^{-1}Z, U_{FTC}) + GL(\overline{\eta} - \eta)$and $\underline{\Pi}_z = G\Gamma(G^{-1}Z, U_{FTC}) - G^+(\underline{\Gamma}(\underline{Z}, \overline{Z}, U_{FTC}) + \underline{d}) + G^-(\overline{\Gamma}(\underline{Z}, \overline{Z}, U_{FTC}) + \overline{d}) + GL(\eta - \underline{\eta})$.
Since E is assumed to be non-negative, $\overline{\Pi}_z$ and $\underline{\Pi}_z$ are non-negative. As a result, for any initial condition $\underline{Z}(0)$ and $\overline{Z}(0)$ chosen such that $\underline{e}_z(0)$ and $\overline{e}_z(0)$ are non-negative, the dynamics of the interval estimation errors $\overline{e}_z$ and $\underline{e}_z$ stay always non-negative for all time t, so the state is bounded as follows:

$$\underline{Z}(t) \leq Z(t) \leq \overline{Z}(t) \tag{49}$$

Moreover, functions $\overline{\Pi}_z$ and $\underline{\Pi}_z$ are globally Lipschitz, and consequently for $\underline{Z} \leq Z \leq \overline{Z}$ and for a chosen submultiplicative norm, there exist positive constants π_{zi} $i = 1, .., 6$ such that:

$$||\overline{\Pi}_z(\underline{Z}, \overline{Z}, U_{FTC})|| \leq \pi_{z1}||\overline{Z} - Z|| + \pi_{z2}||\underline{Z} - Z|| + \pi_{z3} \tag{50}$$

$$||\underline{\Pi}_z(\underline{Z}, \overline{Z}, U_{FTC})|| \leq \pi_{z4}||\overline{Z} - Z|| + \pi_{z5}||\underline{Z} - Z|| + \pi_{z6} \tag{51}$$

Lemma 4.2. *(Zheng, 2016b) Consider that the matrix* $E = G(A-LC)G^{-1}$ *is Hurwitz and Metzler and that the initial state* Z_0 *verifies* $\underline{Z}(0) \leq Z(0) \leq \overline{Z}(0)$*. If there exist positive definite and symmetric matrices* P_2*,* Q_2 *and* δ *such that the following LMI is satisfied:*

$$\begin{bmatrix} E_z^T P_2 + P_2 E_z + \delta P_2^2 + \frac{\gamma_z}{\delta} I + Q_2 & P_2 \\ P_2 & \frac{-I}{\delta} \end{bmatrix} \leq 0, \tag{52}$$

with $E_z = diag(E, E)$ and $\gamma_z = 2\, max(\pi_{z1}^2, \pi_{z2}^2, \pi_{z3}^2, \pi_{z4}^2, \pi_{z5}^2, \pi_{z6}^2)$. Then, the variables $\overline{Z}(t)$ and $\underline{Z}(t)$ are bounded for all the time.
The convergence of the interval observer (48) returns for the difference between the upper and lower state estimates. Thus, the total error is considered as follows:

$$\tilde{Z} = \overline{e}_z + \underline{e}_z = \overline{Z} - \underline{Z} \tag{53}$$

The dynamics of the total error observer is given as follows:

$$\dot{\tilde{Z}} = E\tilde{Z} + \tilde{\Pi}_z \tag{54}$$

with $\tilde{\Pi}_z = \overline{\Pi}_z + \underline{\Pi}_z$.

Lemma 4.3. *(Gouzé, 2000) If gain L is chosen such that the matrix E is asymptotically stable and the total uncertainty vector* $\tilde{\Pi}_z$ *is bounded by a fixed positive vector N, then the interval error* $\tilde{Z} = \overline{Z} - \underline{Z}$ *asymptotically converges to:*

$$\Lambda = -(A - LC)^{-1} N \tag{55}$$

Since N depends on the upper and lower values of m, c, l, I_x, I_y, I_z, d and η, then the size of the estimated sets has to be proportional to model uncertainties. In addition, if there's no uncertainties in the quadrotor model, then Λ converges exponentially to zero. Consequently the lower and the upper trajectories converge toward the current state of the system.
The interval observer design enables obtaining deterministic dynamic intervals containing the real state vector. Accordingly, the centre of the interval can be chosen as a robust state estimation for the uncertain quadrotor system (34) as follows:

$$\hat{X} = \frac{\underline{X} + \overline{X}}{2} \tag{56}$$

Choosing the center of the interval as a robust state estimation has no influence on the trajectory tracking results because any $\hat{X} \in [\underline{X}, \overline{X}]$ can be considered as a guaranteed state estimation for the quadrotor. Generally, the center of the interval is a classic choice used in many studies (Efimov, 2015; Lamouchi, 2017b) which has investigated the control based on the interval observer.

5. Guaranteed tracking control

In this section, based on the interval observer result, a guaranteed tracking control is developed for the quadrotor. Replacing the real state by a robust state estimation defined by equation (56) in the feedback controller (25), the estimated feedback law can be obtained as follows:

$$\hat{v}_i = \ddot{F}_{id} - k_{\alpha i}\dot{\hat{e}}_{ri} - k_{\beta i}\hat{e}_{ri} \quad i = 1, 2, 3, 4, 5, 6. \tag{57}$$

with $\hat{e}_{ri} = \hat{F}_i - F_{id} \quad i = 1, 2, 3, 4, 5, 6.$
When replacing state F by the robust state estimation $\hat{F}$ in the u_{FTC} control, the Guaranteed Flatness-based Tracking Control (GFTC) applied to the uncertain quadrotor (34) system can be obtained as follows:

$$u_{GFTC1} = m\sqrt{(\ddot{F}_{1d} - k_{\alpha 1}\dot{\hat{e}}_{r1} - k_{\beta 1}\hat{e}_{r1})^2 + (\ddot{F}_{2d} - k_{\alpha 2}\dot{\hat{e}}_{r2} - k_{\beta 2}\hat{e}_{r2})^2 + (\ddot{F}_{3d} - k_{\alpha 3}\dot{\hat{e}}_{r3} - k_{\beta 3}\hat{e}_{r3} + g)^2} \tag{58}$$

$$u_{GFTC2} = \frac{Iy}{l}(\ddot{F}_{4d} - k_{\alpha 4}\dot{\hat{e}}_{r4} - k_{\beta 4}\hat{e}_{r4}) - (\dot{F}_{5d}\dot{F}_{6d})\frac{I_z - I_x}{l} \tag{59}$$

$$u_{GFTC3} = \frac{Ix}{l}(\ddot{F}_{5d} - k_{\alpha 5}\dot{\hat{e}}_{r5} - k_{\beta 5}\hat{e}_{r5}) - (\dot{F}_{4d}\dot{F}_{6d})\frac{I_z - I_y}{l} \tag{60}$$

$$u_{GFTC4} = \frac{I_z}{c}(\ddot{F}_{6d} - k_{\alpha 6}\dot{\hat{e}}_{r6} - k_{\beta 6}\hat{e}_{r6}) - (\dot{F}_{4d}\dot{F}_{5d})\frac{I_y - I_x}{c} \tag{61}$$

with $\hat{e}_{r1} = \hat{F}_1 - F_{1d}$, $\hat{e}_{r2} = \hat{F}_2 - F_{2d}$, $\hat{e}_{r3} = \hat{F}_3 - F_{3d}$, $\hat{e}_{r4} = \hat{F}_4 - F_{4d}$, $\hat{e}_{r5} = \hat{F}_5 - F_{5d}$ and $\hat{e}_{r6} = \hat{F}_6 - F_{6d}$.
The guaranteed tracking controller u_{GFTC} differs from the tracking controller u_{FTC} in the way that in u_{GFTC} the robust state estimation $\hat{F}$ is utilized, but in u_{FTC} state F is applied.

Theorem 5.1. *If the nonlinear system (34) is flat and the assumptions* 1, 2, 3 *and* 4 *are satisfied, then it's possible to create a control law based on flatness and an interval observer, which guarantees tracking to a desired reference trajectory in a precise interval despite the existence of unknown but bounded uncertainties.*

Proof. To prove the stability of the (system + controller), let us define the Lyapunov function candidate as follows:

$$V_r = e_r^T P_1 e_r \tag{62}$$

with $e_r = [e_{r1}, \dot{e}_{r1}, e_{r2}, \dot{e}_{r2}, e_{r3}, \dot{e}_{r3}, e_{r4}, \dot{e}_{r4}, e_{r5}, \dot{e}_{r5}, e_{r6}, \dot{e}_{r6}]^T$, and P_1 is a symmetric positive defined matrix. The derivate of Lyapunov function V_r is defined as follows:

$$\dot{V}_r = \dot{e}_r^T P_1 e_r + e_r^T P_1 \dot{e}_r \tag{63}$$

When substituting $\dot{e}_r$ by equation (32) in equation (63), the derivate of Lyapunov function V_r is defined as follows:

$$\dot{V}_r = e_r^T (H_r^T P_1 + P_1 H_r) e_r \tag{64}$$

The gains $k_{\beta i}$ and $k_{\alpha i}$ $i = 1, 2, 3, 5, 6$ are chosen such that matrix H_r is Hurwitz. Thereby, for any symmetric positive definite matrix Q_1, there exists a symmetric positive definite matrix P_1 satisfying the Lyapunov equation:

$$H_r^T P_1 + P_1 H_r = -Q_1 \tag{65}$$

According to (65), the derivative of the Lyapunov function V_r can be written as follows:

$$\dot{V}_r = -e_r^T Q_1 e_r \tag{66}$$

Thus, based on the Lyapunov method, the asymptotic stability for trajectory tracking is deduced.
To prove the stability of the observer, let us recall the proof defined in (Zheng, 2016b), which shows that the variables $\overline{Z}(t)$ and $\underline{Z}(t)$ are bounded. Thus, considering the positive definite quadratic Lyapunov function as follows:

$$V_z = e_z^T P_2 e_z \tag{67}$$

with $e_z = (\overline{e}_z^T, \underline{e}_z^T)^T$ and P_2 is a positive definite symmetric matrix. The observation error system (48) can be rewritten as:

$$\dot{e}_z = E_z e_z + \Pi_z(\underline{Z}, \overline{Z}, U_{FTC}) \tag{68}$$

with $\Pi_z(\underline{Z}, \overline{Z}, U_{FTC}) = [\overline{\Pi}_z^T(\underline{Z}, \overline{Z}, U_{FTC}), \underline{\Pi}_z^T(\underline{Z}, \overline{Z}, U_{FTC})]^T$. Since matrix E is Hurwitz and Metzler, so is the matrix E_z. The derivate of V_z can be defined as follows:

$$\dot{V}_z = e_z^T (E_z^T P_2 + P_2 E_z) e_z + 2 e_z^T P_2 \Pi_z \tag{69}$$

$$\leq e_z^T (E_z^T P_2 + P_2 E_z) e_z + \delta e_z^T P_2^2 e_z + \frac{1}{\delta} ||\Pi_z||^2 \tag{70}$$

According to Corollary 1 defined in (Zheng, 2016b), there exists a positive constant γ_z such that:

$$||\Pi_z||^2 \leq \gamma_z (||e_z||^2 + 1) \tag{71}$$

with $\gamma_z = 2\, max(\pi_{z1}^2, \pi_{z2}^2, \pi_{z3}^2, \pi_{z4}^2, \pi_{z5}^2, \pi_{z6}^2)$. Thus equation (70) can be written as:

$$\dot{V}_z \leq e_z^T (E_z^T P_2 + P_2 E_z + \delta P_2^2 + \frac{\gamma_z}{\delta} I) e_z + \gamma_z \tag{72}$$

Therefore, if there exist positive definite symmetric matrices P_2 and Q_2 and a positive scalar δ such that the LMI defined in (52) is satisfied, then the equation (72) can be defined as follows:

$$\dot{V}_z \leq e_z^T (-Q_2) e_z + \gamma_z \tag{73}$$

This implies that the variables $\underline{Z}(t)$ and $\overline{Z}(t)$ are bounded for all $t_0 \geq 0$.
To prove the global stability of the complete closed-loop system (system + controller + state observer), let us define the Lyapunov function candidate:

$$V = V_r + V_z \tag{74}$$

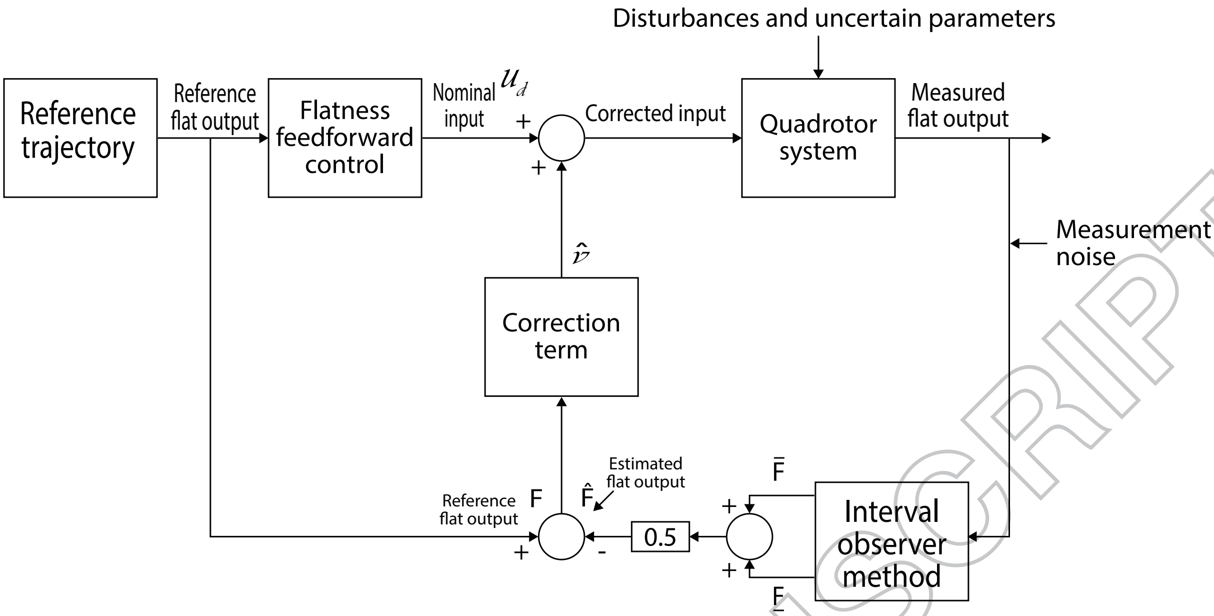

Figure 2. Block diagram of the proposed guaranteed tracking control applied to the quadrotor

It follows from the asymptotic stability of each subsystem that the global asymptotic stability of the complete closed-loop system is guaranteed. Figure 2 depicts the block diagram of the guaranteed tracking control applied to the quadrotor.

6. Simulation and results

In this section, two kinds of simulation are presented to validate the performance obtained by the guaranteed tracking control. Then the quadrotor parameters considered in the simulation are defined as follows: $I_x = I_y = 0.002\ Kg.m^2, I_z = 0.004\ Kg.m^2$, $m = 0.5\ Kg$, $l = 0.2\ m$, $c = 1$, $g = 9.81\ m.s^{-2}$.
Before dealing with the interval observer and the tracking control, it is desired to generate the trajectory that allows the quadrotor to move from an initial state $X(0) = [0,0,0,0,0,0,0,0,0,0,0,0]^T$ to a final one $X(5) = [5,0,11,0,0,0,0,0,0,0,0,0]^T$. In addition, the obtained trajectory must respect the following constraint:

$$-10\ m \leq F_{1d}, F_{2d}, F_{3d} \leq 10\ m, -90^\circ \leq F_{4d}, F_{5d}, F_{6d} \leq 90^\circ\ . \tag{75}$$

To get feasible trajectories, we need to set all the flat outputs as appropriate functions. Several curves can be used to specify the flat output (Fourier series, polynomials, ...). Nowadays, The Bezier curve appears as a powerful tool to generate the reference trajectory for UAV because of its capacity of smoothing and having the advantage of passing through initial and final points while the whole trajectory still lies within the convex constructed by the control points. This paper introduces an eleven-order Bezier curve for each flat output to smooth the path, which can be formulated as follows:

$$F(t) = \sum_{k=1}^{11} B_{k,11}(t) P_k,\ t \in [t_0, t_f] \tag{76}$$

where P_k, $k = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$ are the control parameters of the Bezier curve, which can be chosen based on the initial and final conditions of the flat outputs and their derivatives as well as the trajectory constraint, and $B_{k,11}(t)$ is Bernstein polynomial defined as follows:

$$B_{k,11}(t) = \frac{11!}{k!(11-k)!}(\frac{t_f - t}{t_f - t_0})^{11-k}(\frac{t - t_0}{t_f - t_0})^k \tag{77}$$

To keep the feasibility of the control algorithm, a control input saturation is defined as follows:

$$0.2 \leq u_1 \leq 20\ , -5 \leq u_i \leq 5 \quad i = 2, 3, 4. \tag{78}$$

6.1. Simulation 1

In this simulation, the lower and upper values of the initial state, the uncertain parameters, the sinusoidal wind disturbances and the noise are defined as follows:

$$\underline{X}(0) = [-2, -2, -2, -2, 0, 0, -4, -4, -4, -4, -4, -4]^T, \overline{X}(0) = [4, 4, 4, 4, 2, 4, 8, 8, 8, 8, 8, 8]^T. \tag{79}$$

$$\underline{m} = 0.4\ Kg,\ \overline{m} = 0.6\ Kg,\ \underline{l} = 0.1\ m,\ \overline{l} = 0.25\ m,\ \underline{I}_x = \underline{I}_y = 0.001\ Kg.m^2, \\ \overline{I}_x = \overline{I}_y = 0.003\ Kg.m^2,\ \underline{I}_z = 0.002\ Kg.m^2,\ \overline{I}_z = 0.006\ Kg.m^2,\ \underline{c} = 0.85\ ,\ \overline{c} = 1.15. \tag{80}$$

$$\underline{d}_x = \underline{d}_y = \underline{d}_z = -0.5,\ \underline{d}_\theta = \underline{d}_\phi = d_\psi = -0.2,\ \overline{d}_x = \overline{d}_y = \overline{d}_z = 0.5,\ \overline{d}_\theta = \overline{d}_\phi = d_\psi = 0.2. \tag{81}$$

$$\underline{\eta}_x = \underline{\eta}_y = \underline{\eta}_z = -0.1, \underline{\eta}_\theta = \underline{\eta}_\phi = \eta_\psi = -0.3, \overline{\eta}_x = \overline{\eta}_y = \overline{\eta}_z = 0.4, \overline{\eta}_\theta = \overline{\eta}_\phi = \eta_\psi = 0.2. \tag{82}$$

The controller design parameters are chosen as $\omega_{1c} = \omega_{2c} = \omega_{3c} = 5\ rad/s$ and $\omega_{4c} = \omega_{5c} = \omega_{6c} = 2\ rad/s$. When choosing the matrix observer gain L

$$L = \begin{bmatrix} 18.8356 & -0.1023 & 0 & 0.6839 & 0 & 0 \\ 77.5838 & -1.9963 & 0 & 6.1817 & 0 & 0 \\ -0.1167 & 23.9689 & 0 & -2.2045 & 0 & 0 \\ -2.1960 & 141.8393 & 0 & -26.0802 & 0 & 0 \\ 0 & 0 & 25.9233 & 0 & 0 & -0.3578 \\ 0 & 0 & 158.7713 & 0 & 0 & -5.4602 \\ 0.8732 & -2.1307 & 0 & 20.1956 & 0 & 0 \\ 8.1003 & -25.2926 & 0 & 96.9561 & 0 & 0 \\ 0 & 0 & 0 & 0 & 23.0000 & 0 \\ 0 & 0 & 0 & 0 & 120 & 0 \\ 0 & 0 & -0.2674 & 0 & 0 & 26.0767 \\ 0 & 0 & -4.2518 & 0 & 0 & 154.3302 \end{bmatrix} \tag{83}$$

The Matrix $(A - LC)$ is Hurwitz and has the eigenvalues −6,−7,−8,−9,−10,−11, −12,−13,−14, −15, −16 −17. This choice of eigenvalues makes the observer dynamics faster than the system. Matrix $(A - LC)$ cannot be Metzler for any L. Thus, to get a transformation of coordinates, it is obligatory to create a Metzler and Hurwitz matrix

M_t with the same eigenvalues of $(A - LC)$. According to Lemma 1, the matrix G, chosen such that the matrix $G(A - LC)G^{-1}$ is Metzler and Hurwitz, is defined as follows:

$$G = \begin{bmatrix} G_1 & G_2 \\ G_3 & G_4 \end{bmatrix} \tag{84}$$

where

$$G_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 2.0246 & -0.1265 \\ 0 & 0 & 0 & 0 & -0.8023 & 0.0472 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5.1405 & 0.5712 \\ 0 & 0 & 0 & 0 & 4.9182 & -0.4918 \end{bmatrix} \tag{85}$$

$$G_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.0779 & -0.0049 \\ 0 & 0 & 0 & 0 & 19.3581 & -1.1387 \\ 0 & 0 & 17.1429 & -1.1429 & 0 & 0 \\ 0 & 0 & -17.1429 & 2.1429 & 0 & 0 \\ 0 & 0 & 0 & 0 & -17.9982 & 1.9998 \\ 0 & 0 & 0 & 0 & -1.4378 & 0.1438 \end{bmatrix} \tag{86}$$

$$G_3 = \begin{bmatrix} 0.0526 & -0.0048 & -1.6748 & 0.1523 & 0 & 0 \\ -0.8162 & 0.0680 & 0.5577 & -0.0465 & 0 & 0 \\ 2.9314 & -0.2255 & 1.4168 & -0.1090 & 0 & 0 \\ -0.2334 & 0.0167 & 1.4524 & -0.1037 & 0 & 0 \\ -0.5084 & 0.0847 & 0.0205 & -0.0034 & 0 & 0 \\ -0.4259 & 0.0608 & -0.7726 & 0.1104 & 0 & 0 \end{bmatrix} \tag{87}$$

$$G_4 = \begin{bmatrix} 0.7142 & -0.0649 & 0 & 0 & 0 & 0 \\ 1.3385 & -0.1115 & 0 & 0 & 0 & 0 \\ 1.2564 & -0.0966 & 0 & 0 & 0 & 0 \\ -0.7572 & 0.0541 & 0 & 0 & 0 & 0 \\ 0.1121 & -0.0187 & 0 & 0 & 0 & 0 \\ -1.6640 & 0.2377 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{88}$$

Let us compute the bounding functions $\underline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) = [0, \underline{\Gamma}_{21}, 0, \underline{\Gamma}_{41}, 0, \underline{\Gamma}_{61}, 0, \underline{\Gamma}_{81}, 0, \underline{\Gamma}_{101}, 0, \underline{\Gamma}_{121}]^T$ and $\overline{\Gamma}(\underline{X}, \overline{X}, U_{FTC}) = [0, \overline{\Gamma}_{21}, 0, \overline{\Gamma}_{41}, 0, \overline{\Gamma}_{61}, 0, \overline{\Gamma}_{81}, 0, \overline{\Gamma}_{101}, 0, \overline{\Gamma}_{121}]^T$. Based on the property of $cos(.)$ and $sin(.)$ and the inequality $u_{FTC1} \geq 0$, $\underline{\Gamma}_{21}$, $\underline{\Gamma}_{41}$, $\underline{\Gamma}_{61}$, $\overline{\Gamma}_{21}$, $\overline{\Gamma}_{41}$ and $\overline{\Gamma}_{61}$ are defined as follows:

$$\underline{\Gamma}_{21} = \frac{-2u_{FTC1}}{\underline{m}}, \overline{\Gamma}_{21} = \frac{2u_{FTC1}}{\underline{m}}, \underline{\Gamma}_{41} = \frac{-2u_{FTC1}}{\underline{m}}, \overline{\Gamma}_{41} = \frac{2u_{FTC1}}{\underline{m}}, \underline{\Gamma}_{61} = \frac{-u_{FTC1}}{\underline{m}} - g, \overline{\Gamma}_{61} = \frac{u_{FTC1}}{\underline{m}} - g. \tag{89}$$

To compute the bounding function for Γ_{61}, Γ_{81}, Γ_{101}, defining the following functions:

$\underline{\Delta}_1 = min(\overline{\dot{\phi}}\,\overline{\dot{\psi}}, \underline{\dot{\phi}}\,\overline{\dot{\psi}}, \overline{\dot{\phi}}\,\underline{\dot{\psi}}, \underline{\dot{\phi}}\,\underline{\dot{\psi}})$, $\overline{\Delta}_1 = max(\overline{\dot{\phi}}\,\overline{\dot{\psi}}, \underline{\dot{\phi}}\,\overline{\dot{\psi}}, \overline{\dot{\phi}}\,\underline{\dot{\psi}}, \underline{\dot{\phi}}\,\underline{\dot{\psi}})$,

$\underline{\Delta}_2 = min(\overline{\dot{\theta}}\,\overline{\dot{\psi}}, \underline{\dot{\theta}}\,\overline{\dot{\psi}}, \overline{\dot{\theta}}\,\underline{\dot{\psi}}, \underline{\dot{\theta}}\,\underline{\dot{\psi}})$, $\overline{\Delta}_2 = max(\overline{\dot{\theta}}\,\overline{\dot{\psi}}, \underline{\dot{\theta}}\,\overline{\dot{\psi}}, \overline{\dot{\theta}}\,\underline{\dot{\psi}}, \underline{\dot{\theta}}\,\underline{\dot{\psi}})$,

$\underline{\Delta}_3 = min(\overline{\dot{\theta}}\,\overline{\dot{\phi}}, \underline{\dot{\theta}}\,\overline{\dot{\phi}}, \overline{\dot{\theta}}\,\underline{\dot{\phi}}, \underline{\dot{\theta}}\,\underline{\dot{\phi}})$, $\overline{\Delta}_3 = max(\overline{\dot{\theta}}\,\overline{\dot{\phi}}, \underline{\dot{\theta}}\,\overline{\dot{\phi}}, \overline{\dot{\theta}}\,\underline{\dot{\phi}}, \underline{\dot{\theta}}\,\underline{\dot{\phi}})$.

The bounding function for Γ_{61}, Γ_{81} and Γ_{101} are defined as follows:

$$\underline{\Gamma}_{81} = \begin{cases} \underline{\Delta}_1(\frac{\underline{I}_z - \overline{I}_x}{\underline{I}_y}) \ \text{ if } \underline{\Delta}_1 \geq \ 0 + \frac{\underline{l} \ u_{FTC2}}{\overline{I}_y} & \text{if} \quad u_{FTC2} \geq \ 0 \\ \underline{\Delta}_1(\frac{\overline{I}_z - \underline{I}_x}{\underline{I}_y}) \ \text{ if } \underline{\Delta}_1 < 0 + \frac{\overline{l} \ u_{FTC2}}{\underline{I}_y} & \text{if} \quad u_{FTC2} < \ 0 \end{cases} \tag{90}$$

$$\overline{\Gamma}_{81} = \begin{cases} \overline{\Delta}_1(\frac{\overline{I}_z - \underline{I}_x}{\underline{I}_y}) \ \text{ if } \overline{\Delta}_1 \geq \ 0 + \frac{\overline{l} \ u_{FTC2}}{\underline{I}_y} & \text{if} \quad u_{FTC2} \geq \ 0 \\ \overline{\Delta}_1(\frac{\underline{I}_z - \overline{I}_x}{\underline{I}_y}) \ \text{ if } \overline{\Delta}_1 < 0 + \frac{\underline{l} \ u_{FTC2}}{\overline{I}_y} & \text{if} \quad u_{FTC2} < \ 0 \end{cases} \tag{91}$$

$$\underline{\Gamma}_{101} = \begin{cases} \underline{\Delta}_2(\frac{\underline{I}_z - \overline{I}_y}{\underline{I}_x}) \ \text{ if } \underline{\Delta}_2 \geq \ 0 + \frac{\underline{l} \ u_{FTC3}}{\overline{I}_x} & \text{if} \quad u_{FTC3} \geq \ 0 \\ \underline{\Delta}_2(\frac{\overline{I}_z - \underline{I}_y}{\underline{I}_x}) \ \text{ if } \underline{\Delta}_2 < 0 + \frac{\overline{l} \ u_{FTC3}}{\underline{I}_x} & \text{if} \quad u_{FTC3} < \ 0 \end{cases} \tag{92}$$

$$\overline{\Gamma}_{101} = \begin{cases} \overline{\Delta}_2(\frac{\overline{I}_z - \underline{I}_y}{\underline{I}_x}) \ \text{ if } \overline{\Delta}_2 \geq \ 0 + \frac{\overline{l} \ u_{FTC3}}{\underline{I}_x} & \text{if} \quad u_{FTC3} \geq \ 0 \\ \overline{\Delta}_2(\frac{\underline{I}_z - \overline{I}_y}{\underline{I}_x}) \ \text{ if } \overline{\Delta}_2 < 0 + \frac{\underline{l} \ u_{FTC3}}{\overline{I}_x} & \text{if} \quad u_{FTC3} < \ 0 \end{cases} \tag{93}$$

$$\underline{\Gamma}_{121} = \begin{cases} \underline{\Delta}_3(\frac{\underline{I}_y - \overline{I}_x}{\underline{I}_z}) \ \text{ if } \underline{\Delta}_3 \geq \ 0 + \frac{\underline{c} \ u_{FTC4}}{\overline{I}_z} & \text{if} \quad u_{FTC4} \geq \ 0 \\ \underline{\Delta}_3(\frac{\overline{I}_y - \underline{I}_x}{\underline{I}_z}) \ \text{ if } \underline{\Delta}_3 < 0 + \frac{\overline{c} \ u_{FTC4}}{\underline{I}_z} & \text{if} \quad u_{FTC4} < \ 0 \end{cases} \tag{94}$$

$$\overline{\Gamma}_{121} = \begin{cases} \overline{\Delta}_3(\frac{\overline{I}_y - \underline{I}_x}{\underline{I}_z}) \ \text{ if } \overline{\Delta}_3 \geq \ 0 + \frac{\overline{c} \ u_{FTC4}}{\underline{I}_z} & \text{if} \quad u_{FTC4} \geq \ 0 \\ \overline{\Delta}_3(\frac{\underline{I}_y - \overline{I}_x}{\underline{I}_z}) \ \text{ if } \overline{\Delta}_3 < 0 + \frac{\underline{c} \ u_{FTC4}}{\overline{I}_z} & \text{if} \quad u_{FTC4} < \ 0 \end{cases} \tag{95}$$

Take $\pi_{z1} = \pi_{z2} = \pi_{z3} = \pi_{z4} = \pi_{z5} = \pi_{z6} = 1$, So $\gamma_z = 2$. By solving the LMI defined in (52), $\delta = 1$, $P_2 = diag(P_{22}, P_{22})$, and $Q_2 = diag(Q_{21}, Q_{21})$, where Q_{21} is the identity matrix $12 * 12$, and P_{22} is defined as follows:

$$P_{22} = \begin{bmatrix} 0.1052 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0984 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2369 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2049 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1805 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1613 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1458 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1329 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1222 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1458 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0923 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2806 \end{bmatrix} \tag{96}$$

In order to show the robustness of the proposed control, FTC and GFTC are applied to the uncertain quadrotor system (34) under the same conditions. As a consequence, the quadrotor first starts from an uncertain initial condition $X(0) \in [\underline{X}(0), \overline{X}(0)]$ defined as follows: $X(0) = [1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2]^T$. Second, it is subjected to unknown uncertainties with known defined by equations (80-82).
Figure 3 and Figure 4 depict the following results:
- The red curve represents the reference trajectory of the quadrotor.
- The blue and black curves illustrate the upper and lower state estimations of the uncertain quadrotor system (34) obtained by the interval observer.
- The yellow and green curves show the tracking results when applying FTC and GFTC, respectively, to the uncertain quadrotor system (34).

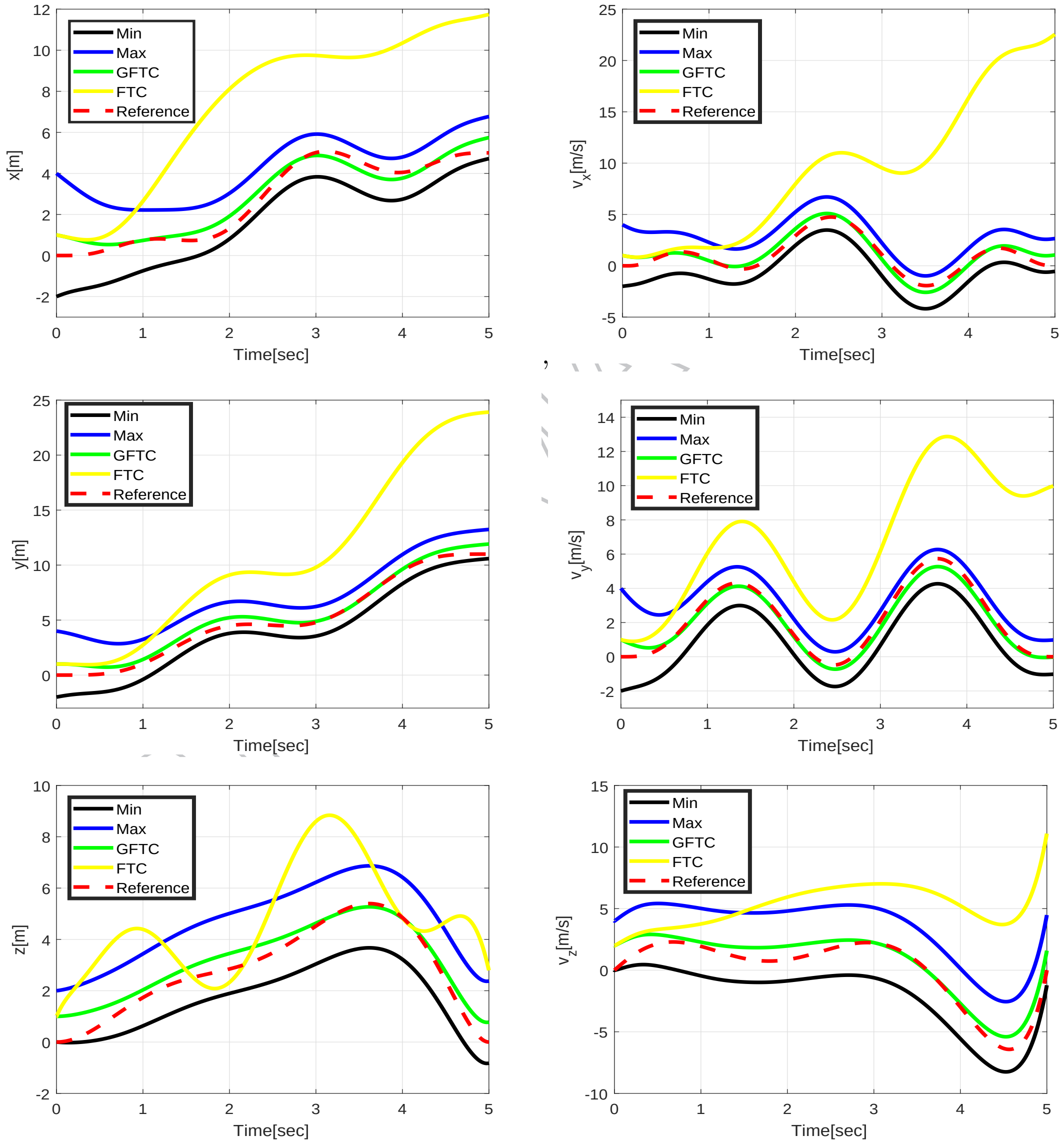

Figure 3. Tracking results for position and velocity of quadrotor subject to unknown uncertainties with known bounds defined by equations (80-82)

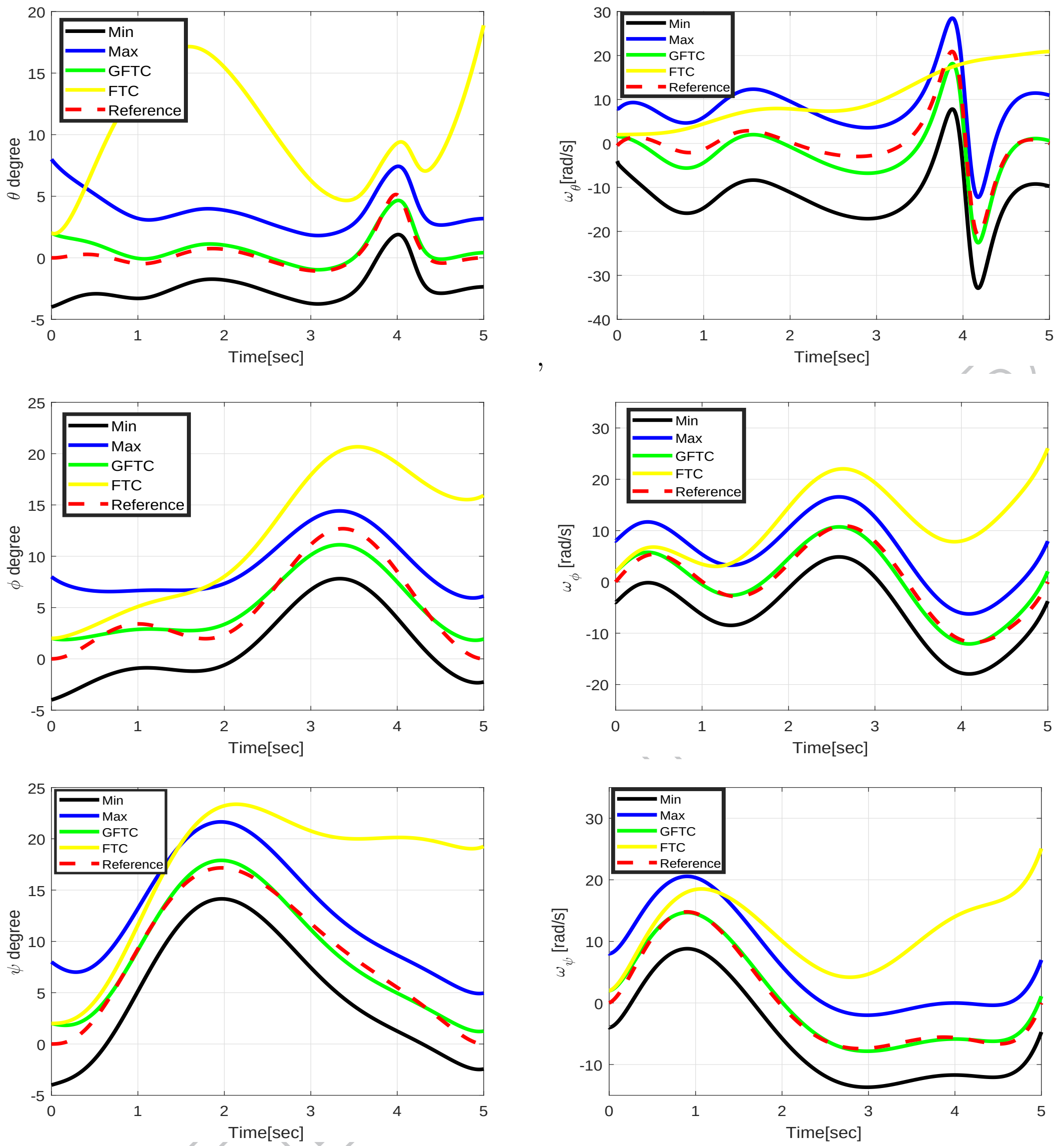

Figure 4. Tracking results for attitude and angular velocity of quadrotor subject to unknown uncertainties with known bounds defined by equations (80-82)

From Figure 3 and Figure 4, it can be observed firstly that choosing a properly gain enables the interval observer to provide a time-varying enclosure, which contains all possible real state vectors of the uncertain quadrotor system (34). This type of observer allows guaranteeing the estimation result of a quadrotor despite the presence of bounded uncertainties. In addition, it can be demonstrated that exploiting the estimation result obtained by the interval observer in the design of GFTC permits the quadrotor to move in a precise interval containing the desired reference trajectory. Whereas, when applying FTC to the uncertain quadrotor system (34), this latter diverges strongly from the desired reference trajectory. As a result, the controllers that are not based on an uncertain model, even if they are feedback controllers, may not work correctly.

6.2. Simulation 2

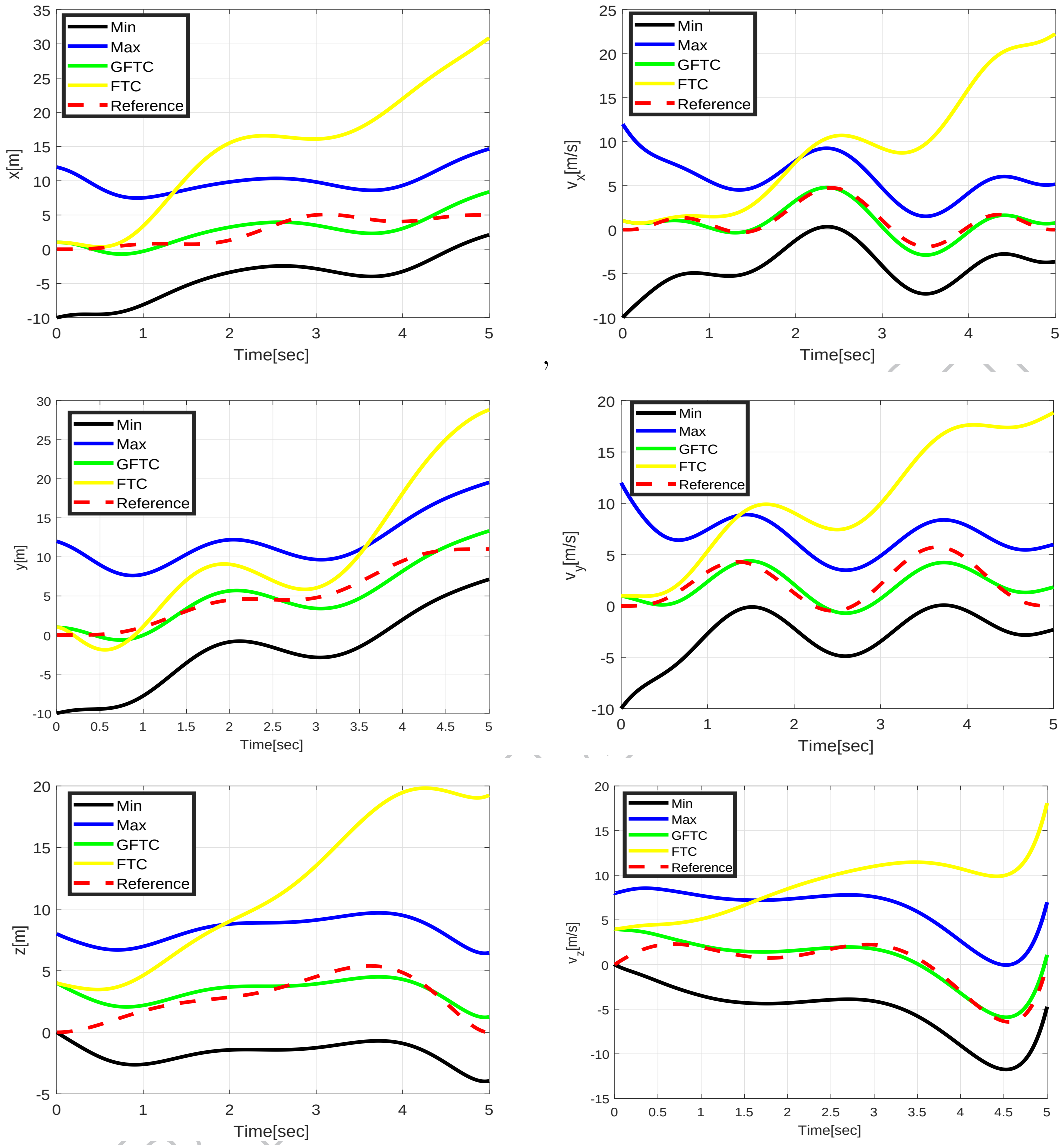

Figure 5. Tracking results for position and velocity of quadrotor subject to unknown uncertainties with known bounds defined by equations (97-99)

A second simulation collection is carried with the same references considered in simulation 1 and with other important values of uncertainty conditions for checking robustness. In this simulation, considering that the quadrotor starts from an uncertain initial condition $X(0) = [1, 1, 1, 1, 4, 4, 5, 5, 5, 5, 5, 5]^T$, it is subjected to unknown uncertainties with known bound defined as follows:

$$\underline{m} = 0.2\ Kg, \overline{m} = 0.8\ Kg, \underline{l} = 0.05\ m, \overline{l} = 0.4\ m, \underline{I}_x = \underline{I}_y = 0.0005\ Kg.m^2, \\ \overline{I}_x = \overline{I}_y = 0.006\ Kg.m^2, \underline{I}_z = 0.001\ Kg.m^2, \overline{I}_z = 0.008\ Kg.m^2, \underline{c} = 0.7\ , \overline{c} = 1.3. \tag{97}$$

$$\underline{d}_x = \underline{d}_y = \underline{d}_z = -2.5, \underline{d}_\theta = \underline{d}_\phi = \underline{d}_\psi = -1.2, \overline{d}_x = \overline{d}_y = \overline{d}_z = 2.5, \overline{d}_\theta = \overline{d}_\phi = \overline{d}_\psi = 1.2. \tag{98}$$

$$\underline{\eta}_x = \underline{\eta}_y = \underline{\eta}_z = -0.5, \underline{\eta}_\theta = \underline{\eta}_\phi = \eta_\psi = -0.8, \overline{\eta}_x = \overline{\eta}_y = \overline{\eta}_z = 0.7, \overline{\eta}_\theta = \overline{\eta}_\phi = \eta_\psi = 0.9. \tag{99}$$

The lower and upper values of the initial state are chosen as follows:

$$\begin{aligned} \underline{X}(0) &= [-10, -10, -10, -10, 0, 0, -10, -10, -10, -10, -10, -10]^T, \\ \overline{X}(0) &= [12, 12, 12, 12, 8, 8, 20, 20, 20, 20, 20, 20]^T. \end{aligned} \tag{100}$$

The feedback gain of the controllers and the observer has been adjusted to obtain a smooth and fast tracking performance. The tracking performances related to simulation 2 are illustrated in Figure 5 and Figure 6 where comparative simulation with FTC and GFTC is also given.

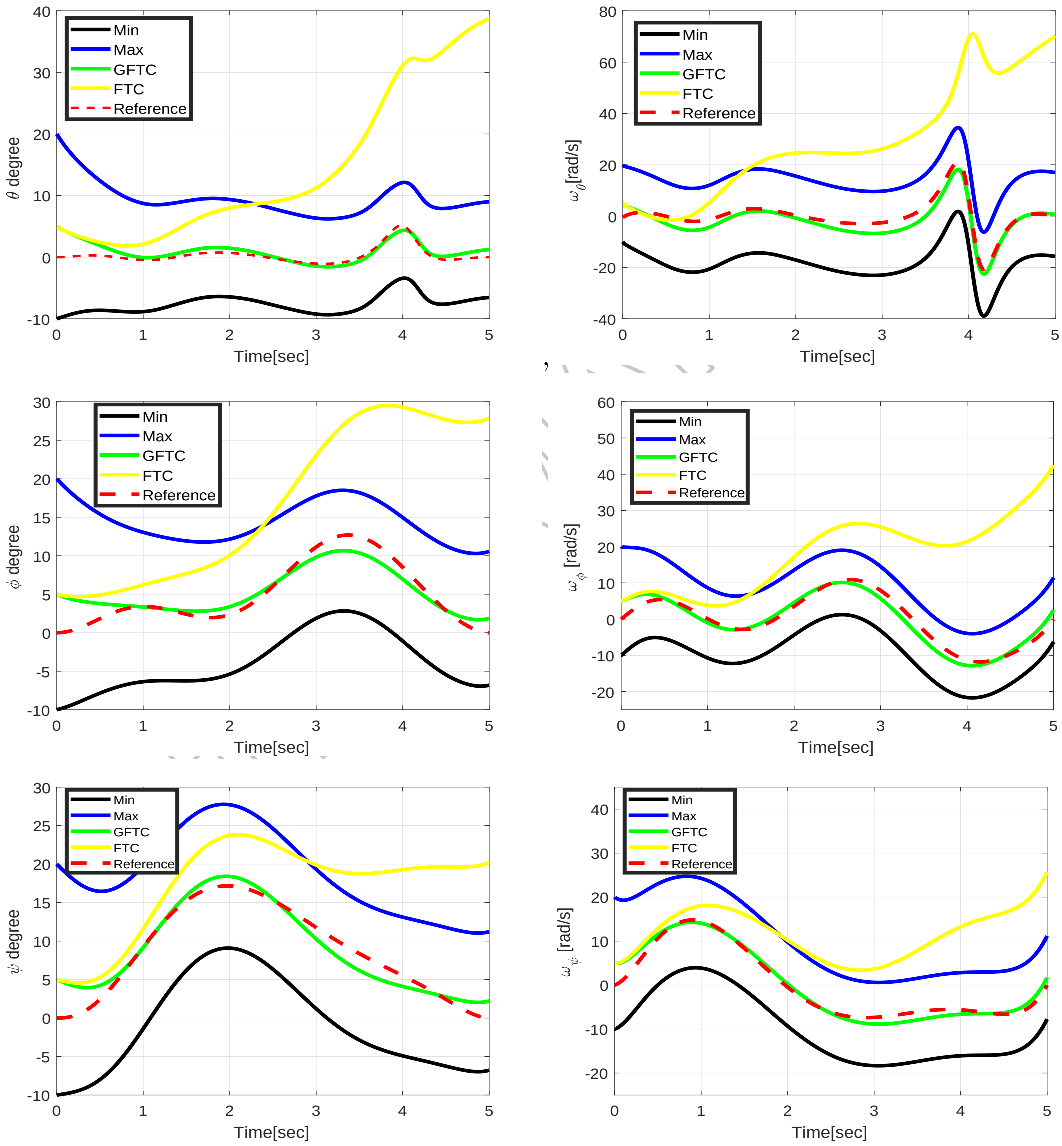

Figure 6. Tracking results for attitude and angular velocity of quadrotor subject to unknown uncertainties with known bounds defined by equations (97-99)

From Figure 5 and Figure 6, the effectiveness of GFTC can be seen compared to FTC. Furthermore, the width of the estimated interval increases compared the interval width obtained in simulation 1, and this is due to the augmentation of the value of uncertainties. Despite this disadvantage, it can be observed that the quadrotor still moves in a precise interval containing the desired reference trajectory. However, the proposed guaranteed tracking controller needs just the limit value of uncertainties affecting the nonlinear system as information to deal with the problem of perturbations. Contrarily, other robust tracking controllers existing in the literature, such as active disturbance rejection control, requires an estimation of lumped uncertainties affecting the system to compensate them, and this concept is very complicated and difficult. Consequently, it can be deduced that the combination between the feedforward flatness control and the interval observer represents a good and new solution for the quadrotor trajectory tracking problem, because it guarantees the tracking performance despite the existence of un-measurable states and unknown uncertainties with important bound values and because it is the first time in the literature that the advantages of interval estimation techniques are exploited in the design of tracking control for an uncertain quadrotor system.

7. Conclusion

In this paper, the problem of tracking trajectories for a quadrotor system with a un-measurable state and unknown but bounded uncertain parameters, disturbances and noise have been studied. To resolve this problem, a Guaranteed Tracking Control (GFTC) has been designed based on linear state feedback control using interval state estimation. The simulation results have shown that the proposed approach guarantees the trajectory tracking for the quadrotor in a precise interval despite the presence of all bounded uncertainties. In future work, observer gains need to be further optimized in order to reduce the effect of uncertainties on the interval estimation accuracy of the quadrotor.

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